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The Reader Collision Problem

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ABSTRACT

We introduce the **reader collision problem**, the problem of allocating frequencies over time to Radio Frequency IDentification (RFID) tag readers such that their interference with one another is minimized. In RFID systems, an RFID tag reader may interfere with the operation of other readers in the system. Interference caused by the operation of an RFID reader is referred to as a reader collision. Reader collisions prevent the colliding readers from communicating with the RFID tags in their respective reading zones. Therefore, reader collisions must be avoided whenever possible to ensure proper and timely communication with tags. The task of preventing reader collisions is referred to as the reader collision problem. We present several centralized global graph-based formulations and variants of the reader collision problem. Additionally, we present and analyze several on-line algorithms to solve the reader collision problem.

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Biography

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Daniel W. Engels received his B.S. from the University at Buffalo, his M.S. from the University of California, Berkeley, and his Ph.D. from the Massachusetts Institute of Technology all in Electrical Engineering and Computer Science. His master's thesis is in the area of computer-aided design for electronic systems, and his doctoral thesis is in the field of theoretical computer science. Dr. Engels joined the Auto-ID Center after obtaining his doctoral degree where he leads the day-to-day research activities of the Center. Dr. Engels' research interests include scheduling theory and applications, real-time system design, distributed and mobile computing, and computer-aided design for embedded systems.

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The Reader Collision Problem

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We introduce the reader collision problem**, the problem of allocating frequencies over time to Radio Frequency IDentification (RFID) tag readers such that their interference with one another is minimized. In RFID systems, an RFID tag reader may interfere with the operation of other readers in the system. Interference caused by the operation of an RFID reader is referred to as a reader collision. Reader collisions prevent the colliding readers from communicating with the RFID tags in their respective reading zones. Therefore, reader collisions must be avoided whenever possible to ensure proper and timely communication with tags. The task of preventing reader collisions is referred to as the reader collision problem. We present several centralized global graph-based formulations and variants of the reader collision problem. Additionally, we present and analyze several on-line algorithms to solve the reader collision problem.**

1. INTRODUCTION

Radio Frequency IDentification (RFID) systems consist of Radio Frequency (RF) tags and networked RF tag readers. The tags themselves are typically comprised of integrated circuits connected to antennae, although simpler analog systems also exist that work without integrated circuits. The readers query the tags for information stored on them; this information can range from static identification numbers to user written data to sensory data. RFID systems are designed to uniquely identify objects. The primary function and capability of an RFID system used within a supply chain is to monitor, in real-time, the location of all tagged objects within the supply chain. Accurate, real-time data about the location of all objects within the supply chain enables the efficient management of the supply chain.

Within segments of the supply chain that exhibit chaotic and sometimes random object placements and movements, a retail environment for example, RFID tag readers must be arranged such that all RFID tags affixed to objects, regardless of where they are within that supply chain segment, can communicate with at least one tag reader. Without such a reader network, accurate real-time information about the location of all objects within that supply chain segment is not possible.

All tag readers have a finite space around them within which they can communicate with tags. This space is referred to as the reader's interrogation zone. A reader arrangement that "covers", or tiles, the entire space within a supply chain segment will have reader redundancy at many locations within that space. That is, there will be locations where the **interrogation zones** of multiple readers intersect. This is because space can only be tiled without redundancy by certain shapes, and the interrogation zones of the readers normally do not conform to these shapes.

Readers whose interrogation zones intersect can interfere with one another, often to the point where neither reader will be able to communicate with any tags located within their respective interrogation zones. Readers may also interfere with one another's operation even if their interrogation zones do not overlap. Such interference is due to the use of radio frequencies for communication, and is similar to the interference experienced in cellular telephone systems. Interference detected by one reader and caused by another reader is referred to as a **reader collision**.

The RFID system must be designed so as to minimize the number and frequency of reader collisions. This can be done by the judicious allocation of frequencies over time to the readers in the RFID system. In this paper, we investigate several problem formulations for performing this allocation.

This paper is organized as follows. In Section 2 we formally define the reader collision problem. Section 3 reviews several concepts that are used throughout this paper. Section 4 presents several graph problem formulations of the reader collision problem. Many of these formulations rely upon the coloring of the graph to determine a solution. In Section 5 we investigate specific graph topologies that model common reader arrangements and present their colorings. Section 6 and Section 7 present and analyze several on-line approximation algorithms to solve the reader collision problem. Finally, Section 8 summarizes our work and presents avenues for future research on the reader collision problem.

2. THE READER COLLISION PROBLEM

2.1. Overview

The reader collision problem models the task of assigning radio frequency spectrum over time to a set of RFID readers. The reader collision problem is a special case of the well-studied frequency assignment problem [11] [4] [12] [7] [10]. The frequency assignment problem models the task of assigning radio frequency spectrum to a set of radio frequency transmitters, such as cellular telephone base stations. In the frequency assignment problem, the mobile devices communicating with the base stations, e.g., cellular telephones, are assumed to be high functionality devices that are able to differentiate between base stations and aid in the communication process with a particular base station. In contrast, the mobile devices communicating with the base stations in the reader collision problem are assumed to be very low functionality RFID tags that are neither capable of differentiating between readers nor capable of otherwise aiding in the communication process with a particular reader.

RFID reader systems encounter many different types of constraints on their operation than do more traditional systems, such as cellular telephone networks, for which the frequency assignment problem has been studied. These different constraints are primarily due to the minimal functionality contained within an RFID tag and the more stringent regulations on the use of the free radio frequency spectrum used by RFID systems.

For the reader collision problem, let $\mathbf{V}$ represent the set of all readers with $\mathbf{v_i} \in \mathbf{V}$ corresponding to reader $\mathbf{i}$, and let $\mathbf{f(v_i, t)}$ denote the continuum of frequencies assigned to $\mathbf{v_i}$ at time $\mathbf{t}$ by the assignment rule $\mathbf{f}$. A feasible assignment of frequencies over time must satisfy certain frequency range and interference constraints, $\mathbf{F}$ and $\mathbf{I}$ respectively. The frequency range constraints may involve either the span of the frequencies assigned or the order (integer number) of frequency channels assigned. The system interference constraints are more variable and often involve the number of resources (both tags and readers) denied service, the percent of time a particular resource is denied service, or the error rate for the combined system.

An optimal assignment of frequencies over time globally minimizes a cost function that depends upon the particular objective of the problem. A common objective that we will consider in this paper is: minimize the time for all readers to communicate given a fixed frequency allotment (i.e., fixed frequency span or fixed order of channels). The system variables we may adjust are frequency and time.

Intuitively, we can assign two readers different times to operate and different frequencies to prevent them from interfering. If possible, we would like to minimize the time span required to let all readers communicate at least once. That is, all readers should be scheduled to communicate as often as possible. Furthermore, given the limited frequency allotment, we would like to reuse frequencies by taking advantage of the spatial nature of radio signal propagation which dictates that signal power is a function of distance from the emitting source, e.g., an RFID reader. That is, readers that are sufficiently separated physically can use the same frequency for communication at the same time without interfering with one another. Frequency reuse may be required if the frequency span is limited. Communicating during time intervals when no other reader is communicating is another method of preserving spectrum. A technique that interleaves communication times on the same frequency is called a Time Division Multiple Access (TDMA) process. A TDMA process is required when the number of readers that can collide is greater than the number of fixed frequencies available for use by the system. More complex communication techniques, such as Code Division Multiple Access (CDMA), are not typically used in RFID systems due to the increased tag complexity and high on-tag power requirements.

2.2. Frequency Range

We often use the term channel instead of frequency when describing a problem or a constraint. Technically, there is a difference. Frequency corresponds to a single wavelength of a continuous sinusoid. A continuum of frequencies in some medium (we consider free space) is a channel with frequency range equal to the difference between the maximum and the minimum frequencies defined for the channel. In RFID systems, the frequency spectrum, **F**, is partitioned into mutually exclusive subintervals of equal length which we define to be **channels**, that is,

$$\mathbf{F} = \{C_1, C_2, \ldots, C_K\} \quad | \quad C_i \cap C_j = \emptyset$$

$$i, j = 1 \ldots K,\ i \neq j,$$

$$C_1 \cup \cdots \cup C_K = \mathbf{F}$$

where the index of each channel is a positive integer, assigned ascending with frequency. The lengths of both the channel interval and the frequency spectrum are determined by the regulations on the frequency spectrum and the operating characteristics of the RFID system.

The bandwidth of a channel is a function of the modulating signal and modulation technique applied. Consequently, channel is the more precise term, however, channel and frequency are typically used interchangeably since the center or **carrier frequency** of a channel often represents the entire channel with the understanding that the carrier frequency has a defined channel bandwidth associated with it.

2.3. Interference

Interference can only be defined with respect to a receiver. That is, any signal is considered to interfere with another signal if and only if it affects the desired signal as seen by the receiver. Consequently, even though frequencies are assigned to transmitters, measurements at receivers define the resulting levels of interference. An RFID reader is both a transmitter and a receiver for its transmissions. An RFID tag uses the reader's own transmission to communicate with the reader. The reader's transmission is typically either reflected or load modulated for the communication. Thus, an RFID tag is simply a receiver and not a transmitter. (A cellular telephone, in contrast, is both a receiver and a transmitter.)

Interference experienced at a receiver is a function of transmitter power, receiver sensitivity, antenna gains, patterns and polarizations, and channel loss. Channel loss is a function primarily of distance, frequency, and weather and is quantified by either minimum acceptable signal to noise power ratio or maximum permissible interference to noise power ratio as measured at the receiver. Most of these interference factors are determined by regulations, are a function of the reader and/or tag design, or are beyond our ability to influence. Consequently, we will consider interference to be a function of frequency and distance.

2.3.1. Reader-to-Reader Frequency Interference

There are two primary types of interference experienced in RFID systems: reader-to-reader frequency interference and multiple reader-to-tag interference. Reader-to-reader frequency interference, or simply frequency interference, occurs when a reader transmits a signal that interferes with the operation of another reader, thus preventing the second reader from communicating with tags in its interrogation zone. This type of interference occurs when the signal transmitted by a reader is of sufficient strength when received at a second reader that the signal masks or jams communication from tags to the second reader. Interrogation zones need not overlap for reader-to-reader frequency interference to occur. The signal from the tag to the reader is extremely weak since tags communicate with readers by either reflecting or loading the reader's own transmission. These signals are easily masked by transmissions from other nearby readers.

We quantify the reader-to-reader frequency interference using the frequency-distance function, $\mathbf{F*D}$ [13]. The frequency distance function models the effective frequency isolation between transmitters. Frequency-distance interference constraints are often easy to construct: compute the distances between pairs of transmitters and receivers and compare them with a minimum distance required to prevent frequency interference given that the pairs operate on the same frequency.

A frequency-distance function for RFID reader systems is calculated as follows. Assume each reader $\mathbf{v_i}$, $\mathbf{i = 1,\ldots,V}$, broadcasts uniformly and omnidirectionally. Define the minimum distance function to be $\mathbf{d(v_i, v_j) = d}$, where $\mathbf{d}$ is the reuse distance, the minimum distance at which a frequency used by both $\mathbf{v_i}$ and $\mathbf{v_j}$ will not cause a reader collision. Let $\mathbf{D(v_i, v_j)}$ be the distance between readers $\mathbf{v_i}$ and $\mathbf{v_j}$, $\mathbf{i \neq j}$. If $\mathbf{D(v_i, v_j) < d}$ and $\mathbf{f(v_i, t) = f(v_j, t)}$, then readers $\mathbf{v_i}$ and $\mathbf{v_j}$ will collide.

2.3.2. Multiple Reader-to-Tag Interference

Multiple reader-to-tag interference, or simply tag interference, occurs when one tag is simultaneously located in the interrogation zones of two or more readers and more than one reader attempts to communicate with that tag at the same time. In this type of interference, each reader may believe it is the only reader communicating with the tag while the tag is in fact communicating with multiple readers. The simple nature of RFID communication can cause the tag to behave and communicate in undesirable ways that interfere with the communicating readers' abilities to communicate with that tag and other tags in their respective interrogation zones.

We quantify the multiple reader-to-tag interference with the interrogation zone-distance function, $\mathbf{Z*D}$. The interrogation zone-distance function is defined to be the effective communication isolation between readers. Interrogation zone-distance interference constraints are often easy to construct, compute the distances between the pairs of readers and compare them with the minimum distance required to prevent interrogation zone interference.

An interrogation zone-distance function for RFID reader systems is calculated as follows. Assume each reader $\mathbf{v_i}$, $\mathbf{i = 1,\ldots,V}$, broadcasts uniformly and omnidirectionally. Define the interrogation zone distance function to be $\mathbf{Z(v_i)}$. Let $\mathbf{D(v_i, v_j)}$ be the distance between readers $\mathbf{v_i}$ and $\mathbf{v_j}$, $\mathbf{i \neq j}$. If $\mathbf{Z(v_i) + Z(v_j) - D(v_i, v_j) > 0}$ and $\mathbf{v_i}$ and $\mathbf{v_j}$ communicate at the same time, then readers $\mathbf{v_i}$ and $\mathbf{v_j}$ will collide.

3. PRELIMINARIES

3.1. Graph Basics

A graph $\mathbf{G = (V, E)}$ is a pair of finite sets where the set **V** contains the **vertices** of the graph and the set **E** contains distinct unordered pairs of vertices called **edges**. Vertices v_i and v_j, $v_i, v_j \in V$, are said to be **adjacent** if they are connected by an edge, that is if $(v_i, v_j) \in E$, and edge (v_i, v_j) is said to be **incident** to vertices v_i and v_j. The **degree** $\deg(v_i)$ of vertex v_i is the number of edges incident upon it. The maximum degree of graph $G = (V, E)$ is defined as $\Delta(G) = \max_{v \in V} \{\deg(v)\}$.

A **multigraph** is a graph in which a pair of vertices may have more than one edge connecting them. If the edge set **E** of a multigraph is partitioned into **K** distinct sets, then we have a multigraph that is a family of **K** graphs $G_1, G_2, \ldots, G_K$ on the same vertex set. We represent such a family of graphs by $G = (V, E_1, E_2, \ldots, E_K)$.

3.2. Simple Graph Coloring and Cliques

The **coloring** of a graph $G = (V, E)$ is defined to be an assignment of colors to the vertices **V** such that no two adjacent vertices (i.e., vertices connected by an edge) have the same color. Colors are typically defined to be the set of nonnegative integers. A coloring that assigns **c** colors to **G** is termed a **c-coloring**. The least number of colors in any such assignment is defined as the **chromatic number** of the graph **G** and is denoted by $\chi(G)$. If readers require only a single channel for communication, then the chromatic number may coincide with the required number of channels.

A **clique** in a graph $G = (V, E)$ is a set of nodes $V_c \subseteq V$ such that any pair of nodes $v_i, v_j \in V_c$ is adjacent in **G**. The order of the largest clique in **G** is called the **clique number** and is denoted by $\omega(G)$. In every graph G

$$\omega(G) \leq \chi(G) \leq \Delta(G) + 1$$

where $\Delta(G)$ denotes the maximum vertex degree of graph **G**.

Simple graph coloring problems, that is problems where the objective is to determine the chromatic number of an arbitrary graph **G**, belong to the hardest class of problems from a complexity theoretic point of view. Computing the chromatic number is **NP**-hard [3] and approximating the chromatic number up to a factor of n^{ϵ} for a constant $\epsilon > 0$ is impossible unless $P = NP$ [9]. However, polynomial time algorithms exist for coloring several families of graphs including bipartite graphs, interval graphs, comparability and cocomparability graphs, planar graphs, and partial **k**-trees for a constant **k** [5].

Many polynomial time solvable graph families are **perfect graphs**. A graph **G** is perfect if and only if for every induced subgraph $G' \subseteq G$, $\chi(G') = \omega(G')$. That is, it is possible to color a perfect graph with the minimum possible number of colors in polynomial time [8]. The text by Jensen and Toft [5] provides a comprehensive overview of graph coloring.

3.3. Set Coloring

A straightforward extension of the simple graph coloring problem is to assign a set of colors, as opposed to a single color, to each vertex. This **set coloring**, or **set assignment**, problem corresponds to problem instances where, for example, an RFID reader requires multiple time intervals to communicate with all tags in its interrogation zone.

Simple set assignment problems can be modeled and solved by coloring a vertex replicated form of the original graph. Given a graph $\mathbf{G = (V, E)}$ with associated demands $\mathbf{r(v_i)}$ at each vertex $\mathbf{v_i \in V}$, the graph $\mathbf{G'}$ is obtained by replacing each vertex $\mathbf{v_i \in V}$ of the graph $\mathbf{G}$ by a clique of size $\mathbf{r(v_i)}$ with each vertex in this clique having edges to the same vertices as $\mathbf{v_i}$ in $\mathbf{G}$. The set coloring of $\mathbf{G}$ is equivalent to the simple graph coloring of $\mathbf{G'}$.

3.4. T-coloring

Additional constraints can be placed on the separation between colors assigned to adjacent vertices. These constraints are given in the form of a set of integers called a T-set. The simple graph coloring problem has a T-set of $\{0\}$. The T-coloring problem is then defined as follows: given a graph $\mathbf{G = (V, E)}$ and a set $\mathbf{T}$ of non-negative integers, a T-coloring of $\mathbf{G}$ is an assignment $\mathbf{f : V \rightarrow Z_+}$ such that if two vertices $\mathbf{v_i}$ and $\mathbf{v_j}$ are adjacent, then $\mathbf{| f(v_i) - f(v_j) | \notin T}$.

3.5. Modeling the Reader Network

The reader collision problem is easily modeled as a graph. The reader network of the RFID system is modeled as a complete undirected multigraph $\mathbf{G_R = (V, E_{R(1)}, E_{R(2)})}$. The set of vertices $\mathbf{V}$ represent the tag readers. The set of edges $\mathbf{E_{R(1)}}$ represent frequency interference between readers. The set of edges $\mathbf{E_{R(2)}}$ represent tag interference between readers. All edges $\mathbf{(v_i, v_j)_k \in E_{R(k)}}$ are assigned a weight $\mathbf{w_{(v_i, v_j)_k}}$. For $\mathbf{k = 1}$, the weight of an edge corresponds to the maximum frequency-distance function $\mathbf{F * D(v_i, v_j)}$ between readers $\mathbf{v_i}$ and $\mathbf{v_j}$. For $\mathbf{k = 2}$, the weight of an edge corresponds to the maximum interrogation zone-distance function $\mathbf{Z * D(v_i, v_j)}$ between readers $\mathbf{v_i}$ and $\mathbf{v_j}$. The weights are not guaranteed to satisfy the triangle inequality.

An interference graph $\mathbf{G = (V, E) = (V, E_1, E_2)}$ may be derived from the reader network multigraph $\mathbf{G_R = (V, E_{R(1)}, E_{R(2)})}$. Vertices in the interference graph still represent tag readers. An edge $\mathbf{(v_i, v_j) \in E}$ exists for each type of interference that two readers can expect from one another. That is, an edge $\mathbf{(v_i, v_j)_k}$ in the interference graph indicates that readers $\mathbf{v_i}$ and $\mathbf{v_j}$ might collide if operated at the same time. Thus, two vertices in the interference graph are adjacent if the signals transmitted by the corresponding readers could interfere, i.e., $\mathbf{(v_i, v_j)_k \in E_k}$ if $\mathbf{w_{(v_i, v_j)_k}}$ is greater than the interference threshold $\mathbf{(k = 1)}$ or the interrogation zones of readers $\mathbf{v_i}$ and $\mathbf{v_j}$ intersect $\mathbf{(k = 2)}$. It is trivial to observe that the **chromatic number** of an interference graph yields a lower bound on the number of channels that are necessary to satisfy channel requirements.

The basic interference graph may be extended to model additional constraints. The most common additional constraints experienced in RFID reader systems are due to multiple time demand at a reader and frequency separation constraints. The requirement that a reader $\mathbf{i}$ be allocated multiple time slots (assuming a discrete modeling of time) is represented by a function $\mathbf{p_i}$ that is associated with vertex $\mathbf{v_i \in V}$. Any allocation of frequency over time that satisfies this time requirement will provide reader $\mathbf{i}$ with $\mathbf{p_i}$ time units to communicate with tags in its interrogation zone. Unless otherwise specified, $\mathbf{p_i = 1, i = 1, \ldots, |V|}$.

Frequency separation constraints may be required when simultaneous communication on two frequencies may cause an unacceptable level of interference. Frequency separation constraints pertain to frequencies allocated to a particular reader and to readers $\mathbf{i}$ and $\mathbf{j}$ such that $\mathbf{(v_i, v_j)_1 \in E_1}$. Frequency separation constraints are easily modeled using $\mathbf{T}$-sets with $\mathbf{T = \{0\}}$ being the default $\mathbf{T}$-set.

3.6. Algorithm Classification

Frequency allocation algorithms can be classified as **static**, where the set of channels and times allocated to a reader cannot change with time, or **dynamic**, where the set of channels and times allocated to a reader may vary with time. The dynamic channel allocation approach is more flexible since it allows adaptation to changes in the network and usage patterns.

A second classification of the algorithms is **off-line**, where the channels and time allocations are made prior to the system beginning execution, or **on-line**, where the channels and time allocations are made while the system is executing. An off-line algorithm always results in a static schedule. An on-line algorithm, in contrast, may result in either a static or a dynamic schedule.

A final classification of the algorithms that we use is based on the location at which the solution is generated. A **centralized**, or **global**, algorithm generates a schedule for all readers in the network, while a **distributed**, or **local**, algorithm allows each reader or cluster of readers to determine their schedule locally. The distributed approach is more robust, while the centralized approach is more likely to lead to an optimal solution.

4. GRAPH FORMULATIONS

The reader collision problem may be formulated and solved using a graph-based algorithm. Let $G = (V, E)$ be a basic interference graph as defined above. We note that the edges $(v_i, v_j)_1 \in E_1$ force the use of multiple frequencies within a single time slot, and the edges $(v_i, v_j)_2 \in E_2$ force the use of multiple time slots regardless of how many frequencies are used. Consequently, frequency interference and tag interference often form orthogonal sets of constraints allowing the corresponding problems to be solved independently.

We first consider problems containing only frequency interference constraints. We then consider problems containing only tag interference constraints. Finally, we consider problems containing both frequency interference and tag interference constraints.

4.1. Frequency Interference

When frequency interference only exists within an RFID reader system, the reader collision problem is equivalent to the co-channel frequency assignment problem which, in turn, is equivalent to the simple graph coloring problem. The simple frequency constrained reader collision problem may be stated as follows,

Instance: $G = (V, E) = (V, E_1)$

Find: $f : V \rightarrow Z_+$, s.t. $(v_i, v_j) \in E \leftrightarrow f(v_i, t) \neq f(v_j, t)$

RFID readers may only communicate on one channel at a time. Therefore, allocating a reader multiple channels does not improve its communication bandwidth.

The co-channel frequency assignment problem was first formalized as a simple graph coloring problem by Hale [4]. Hale considered two objective functions for the co-channel frequency assignment problem, the discrete number of channels used in the solution, called the **order**, and the difference between the highest frequency used and the lowest frequency used in the solution, called the **span**. Clearly, the minimum order of a co-channel problem is simply $\chi(G)$.

When frequency separation constraints are present, the simple graph coloring problem does not sufficiently model the reader collision problem. However, a **T**-coloring of the interference graph $G = (V, E)$ does correspond to a solution of the frequency constrained reader collision problem. Cozzens and Roberts recognized this correlation for the co-channel frequency assignment problem, and they proved several results regarding optimality criteria [2]. Cozzens and Roberts refer to the minimum order for the **T**-coloring of graph **G** as the **T**-order, denoted by $\chi_T(G)$.

THEOREM 1. (Cozzens and Roberts [2])
For any graph G and any T-set such that $0 \in T$,

$$\chi_T(G) = \chi(G).$$

Theorem 1 proves that the order of any **T**-coloring of any graph is just the chromatic number of the graph.

Since RFID systems operate within the free ISM frequency bands, the number of available frequency channels is limited. The exact number of available channels depends upon the local regulations and the design of the RFID system. In many instances the number of available channels is less than the chromatic number of the interference graph. This is particularly true of the lower frequencies used within RFID systems. For example, the 13.56 MHz ISM band extends from 13.553 MHz to 13.567 MHz, but this frequency range may only be divided into a single channel.

When there are not enough channels available to generate a valid solution to the coloring problem, a basic assumption underlying the graph coloring formulation is violated. Namely, the graph coloring formulation assumes that there are enough colors to determine a valid coloring of the interference graph. In these instances, the channels must be multiplexed over time, and the number of time slots used is to be minimized.

We can formulate these reader collision problems as graph coloring problems in which the colors correspond to frequencies allocated over time. Consider the simple reader collision problem in which there are **k** frequency channels available for use by the readers. Solving the graph coloring problem on the interference graph $G = (V, E) = (V, E_1)$ yields a coloring with $c \geq \chi(G)$ colors. Let the colors used be sequentially numbered $1, 2, \ldots, c$. Then, vertices colored with one of $1, 2, \ldots, k$ are scheduled in the first time slot. Vertices colored with one of $k + 1, k + 2, \ldots, 2k$ are scheduled in the second time slot, and so on. The resultant schedule uses $\lceil \frac{c}{k} \rceil$ time slots. Therefore, minimizing the number of colors used to color the interference graph minimizes the number of time slots required to schedule the reader communications.

Now, consider the reader collision problem in which there are **k** frequency channels available for use by the readers and frequency separation constraints exist. **T**-coloring so as to minimize the number of colors used will result in a feasible schedule when the colors are divided into time slots as in the simple reader collision problem.

4.2. Tag Interference

When tag interference only exists within an RFID reader system, the tag reader collision problem is equivalent to a resource constrained scheduling problem. The simple tag constrained reader collision problem may be stated as follows,

Instance: $G = (V, E) = (V, E_1)$

Find: $t : V \rightarrow Z_+$, s.t. $(v_i, v_j) \in E \leftrightarrow t(v_i) \neq t(v_j)$

where $t(v_i)$ denotes the set of times allocated to vertex v_i.

When all readers require the same amount of time to communicate, the tag constrained reader collision problem is equivalent to the simple graph coloring problem where each color corresponds to a unique time slot. A solution to the simple graph coloring problem, therefore, corresponds to a schedule for the readers to communicate with tags in their respective interrogation zones. Thus, minimizing the number of colors minimizes the number of time slots required to allow all readers to communicate.

Next, consider the tag constrained reader collision problem where the readers require multiple time slots to communicate with tags in their respective interrogation zones. Recall that p_i denotes the number of time units required by reader **i** to communicate with all tags in its interrogation zone. Consider the problem where the required time units need not be satisfied by contiguous time slots. Given the interference graph $G = (V, E) = (V, E_2)$ we construct the graph **G'** as follows. Replace each vertex $v_i \in V$ with the independent clique v'_i consisting of p_i vertices. If edge $(v_i, v_j) \in E$, create the set of edges $(v'_i, v'_j) \in E'$ such that each vertex in v'_i has an edge to each vertex in v'_j. Solving the simple graph coloring problem on graph **G'** yields a solution such that a color allocated to a vertex in v'_i is not allocated to a vertex in v'_j if $(v_i, v_j) \in E$. Thus, by interpreting the colors as equal length time slots, an optimal simple graph coloring of **G'** yields an optimal solution to the tag constrained reader collision problem.

Consider the problem where the required time units must be satisfied by contiguous time slots. Solving the simple graph coloring problem on the replicated vertex graph **G'** does not guarantee that the time slots allocated to a reader are contiguous. Therefore, we formulate the reader collision problem as a generalized set coloring problem where the required number of colors $r(v_i) = p_i$ and the allowable span of the colors that are allocated to vertex v_i is p_i.

4.3. Frequency and Tag Interference

Frequency interference and tag interference provide fundamentally different constraints as we saw in the preceding sections. In the general reader collision problem, we may find a feasible solution by considering these two types of interference separately. A generic algorithm for solving the general reader collision problem is as follows. Given the interference graph $G = (V, E) = (V, E_1, E_2)$, solve the tag constrained reader collision problem on $G_2 = (V, E_2)$. The solution to this problem yields a set of groups of vertices, $\{V_1, V_2, \ldots, V_K\}$, where the groupings correspond to a partial order on the vertices $V1 \prec V2 \prec \cdots \prec V_K$. Note that a vertex v_i may belong to a single group only. Frequency interference does not occur between readers in different groups since their operation is separated in time. Therefore, the frequency constrained reader collision problem is solved on each group V_j independently. The combined solution is a feasible allocation of frequencies over time to the readers.

When there are sufficient frequencies to operate all frequency constrained readers at the same time, then the frequency constraints and the tag constraints are orthogonal. Therefore, the reader collision problem may be solved as two separate reader collision problems. Given the interference graph $\mathbf{G = (V, E) = (V, E_1, E_2)}$, solve the tag constrained reader collision problem on $\mathbf{G_2 = (V, E_2)}$ to obtain a time slot allocation for all readers. Solve the frequency constrained reader collision problem on $\mathbf{G_1 = (V, E_1)}$ to obtain a frequency allocation for all readers. (The frequency allocation may also be performed on the vertex groupings as determined by the solution to the tag constrained reader collision problem.) The combination of these two solutions yields a feasible solution to the reader collision problem that allocates frequencies over time to the readers. Furthermore, if each of the sub-solutions is optimal, then the combined solution is optimal.

We note that in this case the tag interference constraints **dominate** the frequency interference constraints. This is due to the time component of the tag interference. Therefore, the problem of coloring graph $\mathbf{G_1}$ can be simplified by removing redundant edges in $\mathbf{E_1}$ such that $\mathbf{E_1 \cap E_2 = \emptyset}$. That is, an edge $\mathbf{(v_i, v_j)_1 \in E_1}$ may be eliminated if edge $\mathbf{(v_i, v_j)_2 \in E_2}$ exists due to tag interference. A feasible solution still results since vertices connected by an edge in $\mathbf{E_2}$ will be allocated different time slots.

In the general reader collision problem we are not guaranteed to find an allocation of frequencies such that all frequency constrained readers may operate at the same time. When the frequency constraints force the use of multiple time slots, the vertex groupings found due to the tag interference provide only a partial order on the reader communication times. The exact communication timings are determined by the combination of the frequency constrained solution and the tag constrained solution.

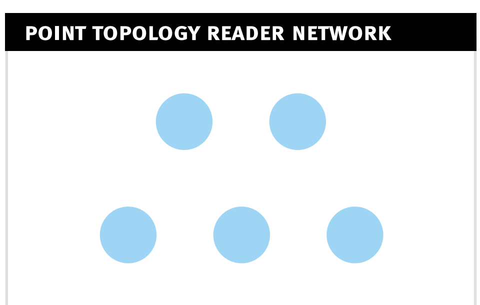
5. SPECIFIC GRAPH TOPOLOGIES

The problem of finding a $\chi(G)$-coloring on an arbitrary graph G is intractable in the general problem instance. However, specific graph topologies and demand patterns recur in RFID reader networks. $\chi(G)$-colorings for these topologies may be precomputed and applied wherever these topologies occur. We examine several topologies and their simple graph colorings in this section.

5.1. Point Topology

A single reader may be used to monitor tags that are within a confined area. For example, readers utilized for point-of-sale transactions, e.g. readers at the check-out counter in a grocery store, will have very confined interrogation zones. These interrogation zones will not overlap with one another to ensure that customers are not incorrectly charged for items. The interference graph $\mathbf{G = (V, E)}$ for the point-of-sale reader network has a point topology. That is, $\mathbf{E = \emptyset}$. A graph with a point topology has $\chi(G) = 1$, and is trivially 1-colored as shown in Figure 1. Readers within a 1-colorable reader network are able to act independently of one another without collisions.

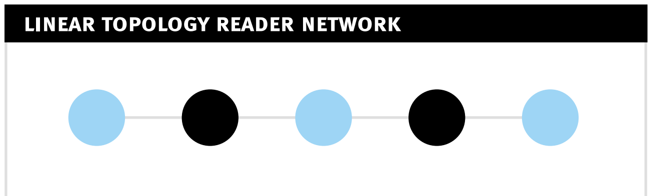
Figure 1: Simple 1-coloring of a point topology.



5.2. Linear Topology

A linear arrangement of readers may be used to cover an area that is long and narrow, a shelf for example. A linear grid pattern, where the distance between the readers at position **i** and position **i + 1** is **x**, is the obvious reader arrangement. This arrangement of readers enables the location of objects on the shelf to be known with a high degree of accuracy. The physically linear arrangement of the readers will often force the corresponding interference graph **G = (V, E)** (or G_1 or G_2) to have a linear topology (due to overlapping interrogation zones, for example). That is, the vertices of the interference graph can be arranged in a line such that the edges connect adjacent vertices only. A graph with a linear topology has $\chi(G) = 2$, and is trivially 2-colored as shown in Figure 2.

Figure 2: Simple 2-coloring of a linear topology.

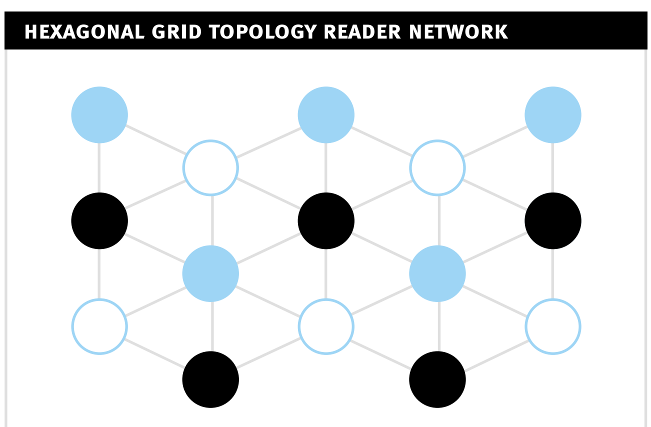


5.3. Area Grid Topology

When a linear arrangement of readers is not sufficient to cover an area, a large expanse of floor for example, a two dimensional arrangement of readers is required. Readers are typically arranged in a simple grid, or “tile”, pattern to ensure coverage of the entire area. Two common tile patterns are hexagonal grids and square grids.

Hexagonal grids are common in cellular phone networks due to the roughly circular communication pattern of the base stations. RFID readers may be designed such that they too have a roughly circular interrogation zone, and when these readers are used to cover an area, a hexagonal grid pattern may be used. In this method, the readers are lined up in columns where the distance between the readers in position i and position **i + 1** within the column is **x**. All columns are separated by a distance **x/2**. The rows are offset such that the distance between the reader in position **i** in column **k** and the reader in position **i** in column **k + 1** is **x**. When interference only occurs between readers that are at most distance **x** apart, the interference graph forms a planar hexagonal grid. A graph with a planar hexagonal grid topology has $\chi(G) = 3$, and is easily 3-colored as shown in Figure 3.

Figure 3: Simple 3-coloring of a planar hexagonal grid topology.



Square grids are another common method of covering an area. In this method, the readers are lined up in rows and in columns such that readers within a column are a distance **x** apart and readers in position **i** in adjacent columns are a distance **x** apart. When interference only occurs between readers that are at most distance **x** apart, the resulting interference graph forms a planar square grid. A graph with a planar square grid topology has $\chi(G) = 2$, and is trivially 2-colored as shown in Figure 4. When interference only occurs between readers that are at most distance $\sqrt{2}x$ apart, the resulting interference graph forms a square grid. A graph with a square grid topology has $\chi(G) = 4$, and is easily 4-colored as shown in Figure 5.

Figure 4: Simple 2-coloring of a planar square grid topology.

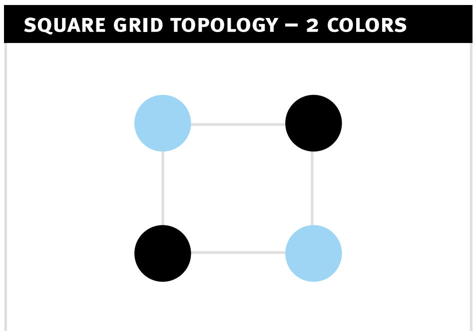
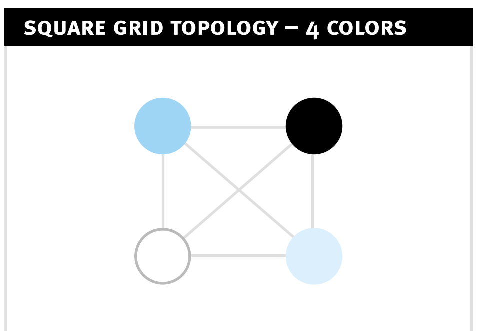


Figure 5: Simple 4-coloring of a square grid topology.



5.4. Three Dimensional Grid Topology

The limited range at which RFID readers can communicate with tags requires them to be arranged in three dimensions such that they cover an entire volume of space. Readers are typically arranged in a simple grid, or "tile", pattern to ensure coverage of the entire volume. We examine a cubic grid pattern here.

In the cubic grid pattern, the readers are arranged in parallel planes a distance **x** apart with each plane containing a square grid with readers a distance **x** apart. When interference only occurs between readers that are at most distance **x** apart, the resulting interference graph forms a simple cubic grid as shown in Figure 6. Each face of the cubes within the simple interference graph exhibits a planar square grid pattern, thus each face is 2-colorable. A graph with a simple cubic grid topology has $\chi(G) = 2$, and is easily 2-colored by offsetting the coloring on adjacent planes as shown in Figure 6.

When interference only occurs between readers that are at most distance $\sqrt{2}x$ apart and within the same plane, the resulting interference graph forms a cubic grid with face interference as shown in Figure 7. Note that no edges are contained within the body of the cube. Each face of the cubes within the

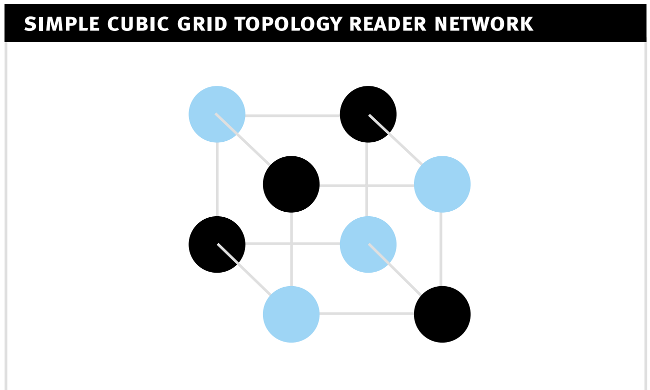


Figure 6: Simple 2-coloring of a simple three dimension square grid topology.

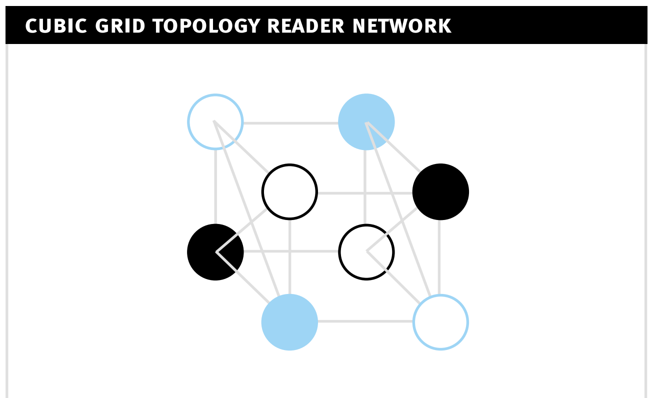


Figure 7: Simple 4-coloring of a three dimension square grid topology with planar interference.

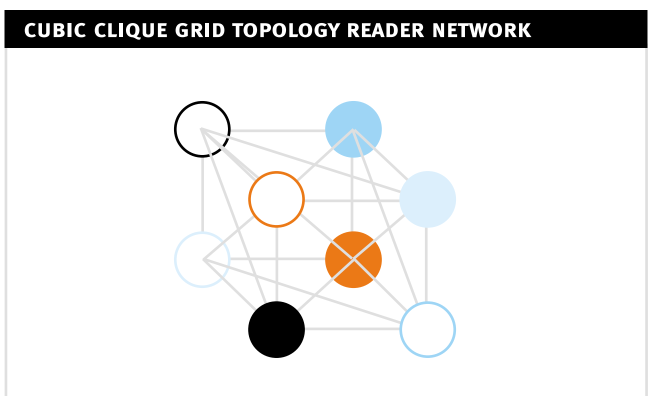


Figure 8: Simple 8-coloring of a three dimension fully connected square grid topology.

interference graph exhibits a (planar) square grid pattern, thus each face is 4-colorable. A graph with a cubic grid topology with face interference has $\chi(G) = 4$, and is easily 4-colored as shown in Figure 7.

When interference only occurs between readers that are at most distance $\sqrt{2}x$ apart regardless of what plane they are in, the resulting interference graph forms a grid of cube cliques as shown in Figure 8. A graph with a cubic clique grid topology has $\chi(G) = 8$, and is easily 8-colored as shown in Figure 8.

6. CENTRALIZED APPROXIMATE ALGORITHMS

The problem formulations in the preceding sections rely upon the solution of a graph coloring problem. The solutions to these problem formulations yield static, globally controlled allocations of frequency over time. We now consider non graph-based problem formulations and algorithms that yield deterministic allocations of frequency over time that are not guaranteed to be optimal.

Consider the following operation of a reader system. Let each reader be a user in the RFID system. Before communicating with tags in its interrogation zone, a reader **i** sends a communication request r_j to the centralized controller at time t_j. The request r_j requests the use of a communication channel for the specified duration p_i. The centralized controller either accepts or rejects the request immediately. The centralized controller accepts requests by allocating the requesting reader a channel until time $t_j + p_i$. The reader uses the channel for the requested duration, and then relinquishes it for use by other readers. Requests are accepted such that no reader collisions are experienced by any reader. Readers whose requests are rejected may resubmit their requests after a random amount of time. The objective is to minimize the number of rejected requests.

Consider the following centralized greedy algorithm CenGreedy: Given a request sequence from all readers $\sigma_r = \{(r_1, t_1), (r_2, t_2), \dots, (r_N, t_N)\}$, with $t_1 \leq t_2 \leq \cdots \leq t_N$. CenGreedy considers the request sequence in order, and accepts request r_j from reader **i** at time t_j if assigning a channel to reader **i** until time $p_i + t_j$ does not cause a reader collision with the already allocated frequencies over time.

THEOREM 2.
Algorithm CenGreedy is $K\Delta(G)$-competitive for any fixed integer $K \geq p_i$, for all readers i in the network.

PROOF.
Let A_{OPT} be the set of requests accepted by the optimal off-line algorithm. Let R_{OPT} be the set of requests rejected by the optimal off-line algorithm. Similarly, let $A_{CenGreedy}$ be the set of requests accepted by the greedy on-line algorithm CenGreedy, and let $R_{CenGreedy}$ be the set of requests rejected by the centralized greedy algorithm CenGreedy. Let $a_{OPT} = |A_{OPT}|$, $r_{OPT} = |R_{OPT}|$, $a_{CenGreedy} = |A_{CenGreedy}|$, and $r_{CenGreedy} = |R_{CenGreedy}|$. Note that $N = a_{OPT} + r_{OPT} = a_{CenGreedy} + r_{CenGreedy}$ where **N** is the size of the input request sequence σ. Finally, let $p_i \leq K$, $i = 1, 2, \dots, |V|$, where **V** is the set of readers.

Suppose $a_{CenGreedy} = k_1 + k_2$ where k_1 is the cardinality of the "optimal" set

$$k_1 = |A_{CenGreedy} \cap A_{OPT}|$$

and k_2 is the cardinality of the "non-optimal" set

$$k_2 = |A_{CenGreedy} \cap R_{OPT}|.$$

The k_1 accepted "optimal" requests may cause the rejection of at most $r_{OPT} - k_2$ other requests. This happens because any "optimal" request can cause the rejection of all the requests that are in R_{OPT} except the k_2 accepted by CenGreedy. Note that an "optimal" request cannot cause the rejection of any other request in A_{OPT}.

CenGreedy can reject at most $k_2K\Delta(G)$ requests because of the k_2 "non-optimal" accepted requests. (Recall that **G** is the interference graph for the reader network.) This holds because any "non-optimal" request may cause the rejection of at most $K\Delta(G)$ "optimal" requests (requests that are in A_{OPT}). Since the total number of requests rejected by CenGreedy is the sum of the requests rejected due to the $k_1 + k_2$ accepted requests, we have

$$r_{CenGreedy} \leq r_{OPT} - k_2 + k_2 K\Delta(G).$$

Since $a_{CenGreedy} = N - r_{CenGreedy}$ it follows that

$$a_{CenGreedy} \geq N - r_{OPT} + k_2 - k_2 K\Delta(G).$$

$$a_{CenGreedy} \geq a_{OPT} - k_2(K\Delta(G) - 1).$$

$$a_{CenGreedy} \geq a_{OPT} - (a_{CenGreedy} - k_1)(K\Delta(G) - 1).$$

$$a_{CenGreedy} K\Delta(G) \geq a_{OPT}.$$

$$a_{OPT} / a_{CenGreedy} \leq K\Delta(G).$$

Therefore, CenGreedy is $K\Delta(G)$-competitive. ■

The CenGreedy algorithm may be implemented as either an off-line algorithm, if the request sequence σ_r is known in advance, or an on-line algorithm. As an on-line algorithm, CenGreedy enables the reader system to dynamically adjust to changing system topologies and required reader communication rates.

We have introduced the notion that a request for service may be rejected with the understanding that the reader whose request was rejected will issue another request at some time in the future. The ability of an algorithm to reject some requests may result in some readers never being allocated time to communicate even though the algorithm itself is considered to be reasonably good at minimizing the total number of rejected requests. We illustrate this point by giving a randomized 4-competitive channel scheduling algorithm for planar interference graphs.

Our randomized algorithm classifies the requests into a number of classes. The algorithm then randomly selects one of the classes and considers only requests that belong to the selected class, rejecting all other requests. We classify our requests according to which vertices in G are making the requests. Towards this end, a preprocessing of the graph is done that produces a partition of the vertices into a number of classes. These classes may be obtained, for example, by coloring the frequency constrained interference graph $G_1 = (V, E_1)$.

For the case of planar graphs, the classification algorithm is a 4-coloring algorithm (solvable in polynomial time [5]) that partitions the graph G_1 into 4 classes (or colors). The randomized algorithm RanCenGreedy selects uniformly at random one of the four classes and considers only requests from the vertices in that class. Allocations of frequency over time are given to the chosen class as in the CenGreedy algorithm.

THEOREM 3.
Algorithm RanCenGreedy for planar graphs is 4-competitive against an oblivious adversary.

PROOF.
Let $A_{(OPT,i)}$ and $A_{(RCG,i)}$ be the set of accepted requests of the optimal off-line algorithm and the greedy algorithm RanCenGreedy, respectively, restricted to requests from the vertices of class i, $i \in \{1, 2, 3, 4\}$. Since there is no edge in **E** connecting two vertices of the same class, $A_{(OPT,i)} = A_{(RCG,i)}$. Let A_{OPT} be the set of requests accepted by the optimal off-line solution in the whole network. Since each request accepted by the optimal algorithm belongs to some class,

$$|A_{OPT}| \leq \Sigma_{i=1}^{4} |A_{(OPT,i)}|.$$

The RanCenGreedy algorithm selects uniformly at random one among the four classes and obtains the optimal benefit for that class. Therefore, the expected benefit of the RanCenGreedy algorithm is

$$1/4 \cdot |A_{OPT}| \leq \Sigma_{i=1}^{4} 1/4 \cdot |A_{(OPT,i)}|.$$

It follows that the RanCenGreedy algorithm is 4-competitive. ■

7. LOCAL APPROXIMATE ALGORITHMS

While the notion of rejecting a request for service in a centralized algorithm is not common, it has found widespread use in distributed algorithms, particularly in communications. The Aloha protocol [1] and its variants are examples of such algorithms. In a common variant, requests for communication are rejected if they would cause a collision, that is if the requested channel is already being used. The requests are resubmitted quickly in the case of the channel being in use, and are resubmitted after a random delay if a collision is detected after communication has commenced.

The distributed, uncoordinated nature of the mechanism of random resubmission delay, or **random backoff**, is amenable to many distributed local algorithms. We utilize the random backoff mechanism, as well as the basic notions of listening for ongoing communications and for collisions with one's own communication in the distributed on-line versions of the CenGreedy and the RanCenGreedy algorithms. Each reader in the RFID system will execute the on-line algorithm.

Consider the following distributed greedy algorithm DisGreedy: Given a request sequence $\sigma_i = \{(r_1, t_1), (r_2, t_2), \ldots, (r_N, t_{N_1})\}$, with $t_1 \leq t_2 \leq \cdots \leq t_N$ at reader i. DisGreedy considers the request sequence in order, and accepts request r_j at time t_j if assigning a channel to reader i does not cause a reader collision with the already allocated frequencies over time, otherwise, request r_j is rejected. DisGreedy determines whether or not a collision will occur by listening to any existing communications in the vicinity of reader i. DisGreedy aborts the communication of reader i if a reader collision is detected during the communication.

A randomized version of communication abortion may be used instead of simply aborting upon the detection of a reader collision. When a reader collision is detected, with probability **P[attempt]** reader i continues to attempt communication, and with probability **1 – P[attempt]** aborts communication. By continuously monitoring for reader collisions we are assured that eventually either reader i will continue communicating without further reader collisions or reader i will abort communication. Distributed greedy algorithms that use this randomized abortion method are referred to as RanDisGreedy algorithms.

Given that either a DisGreedy or RanDisGreedy algorithm is used to either accept or reject reader requests, the request sequences must be generated by the readers. The basic Aloha protocol described earlier is one method of generating the request resubmission sequence. The initial requests may be generated at rate λ, where the rate may be any function including deterministic (e.g., periodic), uniformly at random, and Poisson.

Let's analyze the RanDisGreedy algorithm when it is used to solve the **Remote Control Problem**. Regulations often encourage readers to **frequency hop**. That is, within the frequency range the readers are operating, the readers may emit more power (thereby increasing the size of their interrogation zones) if they do not communicate on any one channel within that frequency range more than a specified fraction of the time. Thus, the readers will need to frequently change the channel they are using for communication. The Remote Control Problem is to minimize the number of reader collisions due to frequency collisions as the readers frequency hop.

Let's analyze the purely random frequency hopping solution to the Remote Control Problem. Each reader implements the RanDisGreedy algorithm with **P[attempt] = 0**, and all requests are for a communication duration of one time unit. Assume that time is slotted with all slots equal to one time unit. RanDisGreedy will always accept a request since no reader is communicating at the beginning of a time slot. However, requests will be rejected later if reader collisions are detected. The readers resubmit a rejected request at the beginning of the next time slot.

Let **c** be the number of channels that all readers may communicate in. Consider **n** readers that form a clique in the interference graph, and the next requested channel is chosen uniformly at random. The probability that these readers randomly choose channels such that no collisions occur is

Formula 1

$$p_{\text{no collisions}} = c! / c^n (c - n)!$$

where c^n is the number of possible channel assignments, and $c!/c^n(c - n)!$ is the number of channel assignments for which distinct channels are chosen by the readers. For the case of 50 channels (feasible at 915 MHz in the United States of America) and 27 readers (think 3-D grid with three readers along each axis), this corresponds to a probability of 0.000157 that no reader collisions occur within the clique assuming all channels are equally likely.

Given that a reader has chosen channel **i**, the probability that at least one of the other **n – 1** readers also chooses that channel is given by:

Formula 2

$$P_{n-1}[i] = \sum_{k=1}^{n-1} \binom{n-1}{k} p^k (1 - p)^{n-k-1}$$

where **p** is the probability that a reader chooses channel **i**. The Binomial Probability Law

Formula 3

$$p_n(k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

indicates the probability of **k** successes on **n** trials. A success in this context corresponds to a reader choosing channel **i** for communication. For the case of 50 channels and 27 readers, this corresponds to a probability of 0.409 assuming all channels are equally likely to be chosen.

Thus, the probability that any particular reader **x** experiences a reader collision is

Formula 4

$$P_x[\text{sees collision}] = P_x[\text{channel} = i] P_{n-1}[i]$$

where P_x**[channel = i] = p** is the probability that reader **x** chooses channel **i**. For the case of 50 channels, 27 readers, and channels are chosen uniformly at random, the probability that a particular reader experiences a reader collision on a particular frequency hop is 0.00817. That is, a particular reader experiences reader collisions due to frequency interference less than one percent of the time.

Let's continue viewing the world from a reader's perspective. Due to signal propagations, it is quite likely reader **i** has degree $\deg(v_i) > c$. The **c** frequency channels that a reader **i** may use to communicate look like a set of **c** servers. Let λ be the average arrival rate, that is the rate at which readers adjacent to **i** (including **i**) in the interference graph $G_1 = (V, E_1)$ attempt to communicate with tags in their respective interrogation zones. Let all readers require time $1/\mu$ on average to complete their communication. If the arrivals are a Poison process with rate λ then the frequency channels around reader **i** may be modeled as an **M | G | m | m** system. Thus, the probability that a reader adjacent to **i**, when it attempts to communicate with the tags in its interrogation zone, finds the frequency channels around i to be busy, and therefore must wait to communicate is given by the Erlang B formula [6],

Formula 5

$$p = ((\lambda/\mu)^c/c!)/(\Sigma_{k=0}^{c}(\lambda/\mu)^k/k)!).$$

8. CONCLUSIONS AND FUTURE WORK

In many ways, the reader collision problem is simpler than the frequency assignment problem. Thus, much of the previous work on the frequency assignment problem is applicable to the reader collision problem. However, the time and frequency usage constraints on the readers and the RFID system have a more significant impact on the performance of the RFID system than more traditional systems modeled by the frequency assignment problem. We have explored some of the consequences of the RFID system specific time and frequency constraints. Most notably, the phenomenon of tag interference is specific to the reader collision problem and is not found in the previously studied frequency assignment problems.

Many reader systems are static, that is the system configuration and usage patterns change infrequently. Thus, global off-line algorithms can yield optimal allocations of frequency over time. We have formulated many of these global problems as graph coloring problems on arbitrary interference graphs. The topology of the reader networks may enable more efficient formulations and algorithms. Specific reader network topologies need to be examined to determine if this is indeed possible. In addition, regulations may require more stringent constraints on the functionality of a centralized global algorithm than we have considered here. The reader collision problem must be investigated with these additional constraints.

Regulations in some parts of the world may prevent the use of centrally controlled RFID reader systems. In these countries, the readers must act independently to secure their required communication time. Therefore, distributed on-line algorithms only may be used within the RFID reader systems. In general, these distributed on-line algorithms yield suboptimal allocations of frequency over time. As we proved for specific cases, the behavior of some of these algorithms may be bounded. Theoretical and empirical analysis must be performed to develop and evaluate these algorithms for realistic operating conditions.

Additional variables may also be considered within the reader collision problem formulations. For example, the readers need not communicate at the maximally allowed power levels at all times. Varying the signal power from a reader varies the interrogation zone and the frequency and tag interference experienced by and caused by that reader. By allowing the power to be a variable to be solved for in the reader collision problem formulations, a solution may be found allowing the readers to communicate frequently with the tags close to them and infrequently with tags further away.

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