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REGULAR ARTICLE

Improving the performance of retail stores subject to execution errors: coordination versus RFID technology

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Abstract This paper analyzes a Newsvendor type inventory model in which a manufacturer sells a single product to a retailer store whose inventory is subject to errors stemming from execution problems. Hence, within the store, all of the products are not available on shelf for sales either because the replenishment of the shelf from the backroom is subject to execution errors that mainly result in products lost in the backroom or products misplaced on the other shelves of the store. We compare two situations: in the first situation, the two supply chain actors are aware of errors and optimize their ordering decisions by taking into account this issue. The second situation deals with the case where an advanced automatic identification system such as the Radio Frequency Identification technology is deployed in order to eliminate errors. Each situation is developed for three scenarios: in the centralized scenario, we consider a single decision-maker who is concerned with maximizing the entire supply chain's profit; in the decentralized uncoordinated scenario, the retailer and the manufacturer act as different parties and do not cooperate. The third scenario is

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the decentralized coordinated scenario where we give conditions for coordinating the channel by designing a buyback contract.

Keywords Newsvendor model · Execution errors · Misplacement · RFID technology · Supply chain coordination

1 Introduction

Although advanced inventory control policies have been developed for almost a century, the occurrence of out-of-stocks is still a significant issue in the retail supply chain. In fact, out of shelf rates vary wildly among retailers and their outlets depend on a variety of factors, but the average out of stock rate falls in the range of 5–10%. This is confirmed by surveys conducted by Gruen et al. (2002) where it is also estimated that the overall out of stock rate is about 8.3% leading to 4% of lost of sales for a typical retailer. Berger (2003) also reports that out of stock rates between 10 and 15% are not unusual in most of European countries.

In investigations exploring the reasons leading to out of stocks, several factors are identified (Gruen et al. 2002; Vuyk 2003): (i) Retail store ordering and forecasting problems, i.e. the ordered quantity is not enough to meet the observed consumer demand, (ii) Execution errors stemming from store shelving and replenishment practices where products ordered are in the store but not on the right shelf. The magnitude of these errors depends on shelf-space allocation strategies, shelf-replenishment frequencies, store personnel capacity, etc. (iii) Factors related to the reliability of the supply system, i.e. the quantity received from supplier does not correspond to the quantity ordered.

Our paper deals with the second factor: we consider a retail store where all products ordered from the supply system are received but one part is not available on shelf due to misplacement type errors. According to Chappell et al. (2003), there are several sources generating misplacement errors such as: (i) consumers picking up products and putting them down in another location, (ii) clerks not storing products on the correct shelf at the right time and (iii) clerks losing products in the backroom.

Misplacement errors are among the other types of factors such as theft, damage, transaction errors, etc... that lead to inefficiencies in operations. The literature dealing with such execution errors and their consequences on supply chain operations is growing. One of the particular point the existing investigations focus on is the issue of inventory data inaccuracies. Among them, DeHoratius and Raman (2004) explore the factors affecting inventory record inaccuracy. They found out that the record inaccuracy increases with sales, the number of stages in the supply chain, product variety, and the number of days elapsed since the last inventory audit. Ton and Raman (2004) also examine empirically the issue of misplaced products in retail stores. Investigations that examine the inaccuracy issue in a quantitative way are still rare. Among them, Sahin (2004) provides a comprehensive analysis of potential errors (including misplacement type errors) that may occur within an inventory system with a special focus on reasons why mismatches occur between the physical flow and the information flow representing it. She builds a general Newsvendor framework to model the impact of errors in order to quantify the cost penalty they generate and evaluates if the implementation of an advanced data capture technology such as RFID is cost justified. Gaukler et al. (2003) investigate the effects of the RFID technology within a retail supply chain. They build a simple decentralized Newsvendor model that takes into account the non efficiency of the replenishment process from the backroom to the shelf within the retail store. The investigation of Kang and Gershwin (2005) considers the inaccuracy caused by shrinkage type error and its impact on inventory management through a simulation study. The authors illustrate how shrinkage increases lost sales and results in an indirect cost of losing customers (due to unexpected out of stocks) in addition to the direct cost of losing inventory. The objective is to illustrate the effect of shrinkage on lost sales through simulation. The investigation of Fleisch and Tellkamp (2005) simulates a three echelon supply chain with one product in which end customer demand is exchanged between the echelons. The authors study the relationship between inventory inaccuracy and performance in a retail supply chain. Mosconi et al. (2004) propose a mathematical model that describes how inventory records become inaccurate over time at retail. the proposed model captures the interaction between inventory record inaccuracy and the variability in the scanning and receiving processes, the amount of inventory on hand, and the level of demand. Kök and Shang (2004) examine policies for triggering inventory audits in a system subject to inventory inaccuracies stemming from transaction errors. The authors study a dynamic programming formulation where the number of days since the last audit serves as a sufficient statistic for the distribution of record error. The investigation of Uckun et al. (2005) focus on the decision of the optimal investment levels in order to decrease the inventory inaccuracy in a two-level supply chain consisting of a supplier and a retailer. The investigation of DeHoratius et al. (2005) considers a periodic review inventory process with unobserved lost sales, and models inventory record inaccuracy through an "invisible" demand process that is reflected in updates of physical inventory but not recorded inventory. The authors propose inventory management tools that account for record errors using a bayesian updating of error distribution. Atali et al. (2005) characterize factors that lead to inventory discrepancy. Some of them result in permanent inventory shrinkage (such as theft and damage); others are temporary and can be recovered by physical inventory audit and returned to inventory (such as misplacement). The final group of factors (such as scanning error) affects only the inventory record and leaves the physical inventory unchanged. Recently Gel et al. (2006) analyze an inventory subject to execution errors in the presence of correction opportunities. In order to evaluate the economic impact of inventory record inaccuracies, the authors simulate a continuous review model and evaluate suboptimalities in cost and customer service that arise as a result of untimely triggering of orders due to inventory record inaccuracies. In particular, they show that the economic impact of inventory record inaccuracies can be significant especially in systems with small order sizes and low reorder levels. In a recent investigation, Lee and Ozer (2005) provide an extended literature review on the impact of the RFID technology on supply chains including a part on the RFID as a lever to reduce inaccuracies.

Our work differs from the existing investigations in that we consider misplacement type execution errors in a retail store which faces a Newsvendor problem. In our paper, we consider two settings: (i) the case of a centralized supply chain structure, (ii) the case of a decentralized supply chain where the manufacturer and the retailer act as different parties.

The first case has been investigated in a previous paper (cf Rekik et al. 2006). Our motivation to consider the second case is to assess how each supply chain actor (the manufacturer and the retailer) will be better-off if misplacement type errors are reduced within the store. We analyze two solutions enabling to achieve this: the first solution consists in deploying the RFID system which is a recent technology that is based on the use of wireless RFID tags and (EPCs) Electronic Product Codes. RFID readers placed at different points within the store enable to detect products automatically (without human intervention) every time items removed and therefore contribute to the elimination of execution errors. The second solution deals with channel coordination, i.e. designing contracts between the manufacturer and the retailer such that their local optimal policies correspond to the global optimal policy of the supply chain. For example in a buy back type contract, the retailer can return the excess order quantity at a partial refund, at the end of the selling season. The idea of coordinating decentralized supply chain using contracts first originated by Pasternack (1985). Larivière (1998) and Cachon (2003) present an excellent overview of decentralized supply chain control. Larivière and Porteus (2001) present further results for contracting under a Newsvendor structure. Jemai et al. (2005) shows that buy back contract generalizes linear transfer payment contracts. In this paper, we build on several of results of the above papers and we use a modified buy back contract to coordinate the channel.

The main questions that this paper addresses are:

1. What is the impact of misplacement type execution errors?
2. Which technology cost make the RFID feasible for both supply chain actors?
3. If we consider an initial situation with errors and no coordination, what is the best strategy to be adopted by supply chain actors: the deployment of the RFID technology or the coordination of the channel?

2 The problem setting

2.1 The modeling of misplaced items in the retail store

We consider a supply chain consisting of one manufacturer and one retailer. The manufacturer produces a single seasonal product which has a unit production cost c and sells it to the retailer. The retailer sells the product in a store to end consumers at a unit price r. It is assumed that, at the end of the season, products can be sold back at a discounted (salvage) price s. The ordering decision of the retailer is made within a one-period Newsvendor framework.

Within this context, we define the parameter θ which reflects the effect of errors on the physical quantity that can be sold to consumers: with a quantity of products Q ordered from the manufacturer is associated two quantities: (i) an amount of products θQ that is on shelf, thus, available to be bought by consumers, and (ii) an amount of products $(1-\theta)Q$ which is not available to buy since stolen or misplaced either in the backroom or on the other shelves. Concerning the portion of products which is not available to buy, one has to distinguish two cases according to the factor that

Table 1 Organization of the paper

	C scenario	DU scenario	DC scenario
Model 1	Section 3.1	Section 3.2	Section 3.3
Model 2	Section 4	Section 4	Section 4

induces it. Indeed, if this stems from theft or perishment, then this quantity cannot be salvaged at the end of the season. If this is induced by execution type errors such as misplacement within the store, the lost quantity $(1-\theta)Q$ would be found at the end of period. We assume that the lost quantity $(1-\theta)Q$ is found at the end of the period and can be discounted. Without loss of generality, we also assume that the cost of finding the misplaced $(1-\theta)Q$ quantity is null. Note, however, that the analysis presented in this paper could easily be modified to deal with theft type errors or to take into account the cost associated with finding a misplaced product (cf Sect. 6). As most of investigations developped within a Newsvendor framework, regarding parameters pertaining to the distribution of demand, we assume that they are exogenous.

2.2 Models and scenarios under study

In order to examine the impact of errors, we consider two models:

1. *Model 1*: the retailer operates with internal errors and both the manufacturer and the retailer know the error parameter θ. Decisions about the ordering quantity are made by taking into account θ.
2. *Model 2*: the RFID technology is deployed within the store to eliminate errors. This model is a slightly modified version of the commonly known Newsvendor problem and takes into consideration the cost of the RFID technology.

The basic Newsvendor problem, which will be called *Model 0*, appears to be a particular case of these two models: in Model 1, if we set $\theta = 1$ we obtain Model 0. In Model 2, if we set the technology cost equal to zero, we also obtain Model 0.

For each model, i.e. Model 1 and Model 2, we examine three scenarios:

1. *The Centralized scenario* (C) where we assume that there is a single decision-maker who is concerned with maximizing the entire chain's profit.
2. *The Decentralized Uncoordinated scenario* (DU) where we consider two decision-makers, the manufacturer and the retailer, and each optimizes his own profit function.
3. *The Decentralized Coordinated scenario* (DC) where the manufacturer and the retailer cooperate in order to make the total expected profit closer to the expected profit associated with the Centralized scenario.

The following table (Table 1) represents the organization of the paper:

Under the centralized scenario, the analysis of models described above and the comparison between them are developed in a previous work (Rekik et al. 2006). In this paper, our first contribution concerns Model 1 which is less investigated in the literature. We examine an inventory system subject to errors and obtain analytical expressions of the optimal policy for both a centralized and a decentralized supply chain structure. Our second contribution concerns the comparison between Model 1 and Model 2 where we provide a sufficient condition on values of model parameters (specially on the cost of the RFID technology) which justifies the deployment of the RFID technology for both supply chain actors. Our third contribution concerns the comparison between two strategies that may enable to improve the performance of a decentralized supply chain in presence of errors in the store. Indeed, we compare two strategies which can be adopted by supply chain actors while being in Model 1 under the DU scenario. The first strategy consists in implementing the RFID technology while staying in the DU scenario. The second strategy consists in improving the performance by coordinating the channel in presence of errors, without using RFID.

2.3 Notations

In the rest of the paper, the following notations are used:

- Q_{ij}: the ordering quantity for Model j $(j = 0, 1, 2)$ under scenario i $(i = C, DU, DC)$
- Q^*_{ij}: the optimal value of Q_{ij} item π^k_{ij}: the expected profit for entity k $(k = M, R)$ in Model j $(j = 0, 1, 2)$ under scenario i $(i = C, DU, DC)$
- w_{ij}: the unit product purchase cost for Model j $(j = 0, 1, 2)$ under scenario i $(i = C, DU, DC)$
- r: the unit product selling price
- s: the unit product salvage price
- c: the unit production cost
- x: the random variable representing demand
- $f(x)(F(x))$: pdf (cdf) characterizing the demand
- μ: the expected demand
- σ: the standard deviation of demand
- ϕ (Φ): the standard normal pdf (cdf)

3 Analysis of Model 1

In Model 1, we assume that both the retailer and the manufacturer are aware of errors and optimize their expected profit function by taking into account the error parameter. The sequence of events in this model is as follows:

1. The order: before the beginning of the selling period, in order to satisfy the store's demand, the retailer orders an amount of products Q_{i1} $(i = C$, DU, DC) from the manufacturer.
2. The total physical inventory: at the beginning of the period the retailer receives the quantity Q_{i1} $(i = C$, DU, DC) to the store.
3. The available to buy quantity: due to internal errors occurring in the store, the quantity on shelf observed by consumers, θQ_{i1} $(i = C$, DU, $-$DC), may be different from the quantity physically available to satisfy demand.

4. The satisfaction of demand: the actual demand x is observed and satisfied from the available to buy quantity.
5. All the unsold quantity (on shelf + misplaced) is discounted at the end of the period.

3.1 Analysis of the Centralized scenario (C1)

We consider a centralized supply chain where both the retailer and the manufacturer are part of the same organization and managed by the same entity. There is a single decision-maker who is concerned with maximizing the entire chain's profit. As a consequence, we can ignore the wholesale price transaction since it is internal. The ordering decision of the centralized decision-maker is made by taking into account θ.

A detailed analysis of this model (and the proofs associated with this section) can be found in Rekik et al. (2006). In this section, we present the main results pertaining to this analysis. The expected profit function of Model 1 under the Centralized scenario is given by:

$$\begin{aligned}\pi_{C1}(Q_{C1}) = (r-c)\mu - (r-c)\int_{x=\theta Q_{C1}}^{+\infty} (x-\theta Q_{C1}) f(x)\,dx \\ -(c-s)\int_{x=0}^{\theta Q_{C1}} (\theta Q_{C1}-x) f(x)\,dx \\ -(c-s)Q_{C1}(1-\theta)\end{aligned} \tag{1}$$

The following proposition states the optimal ordering quantity and the optimal expected profit of Model 1 under the Centralized scenario:

Theorem 1

(a) *The expected profit function is concave in the ordering quantity* Q_{C1}.
(b) *The optimal ordering quantity for Model* 1 *in the Centralized scenario is such that*:

$$\begin{aligned}F(\theta Q^*_{C1}) &= \frac{r\theta + (1-\theta)s - c}{(r-s)\theta} \quad \text{for} \quad \theta \geq \frac{c-s}{r-s} \\ Q^*_{C1} &= 0 \qquad \text{otherwise}\end{aligned} \tag{2}$$

(c) *The optimal expected profit for Model* 1 *in the Centralized scenario is such that*:

$$\begin{aligned}\pi_{C1}(Q^*_{C1}) &= (r-s)\int_{x=0}^{\theta Q^*_{C1}} x f(x) dx \quad \text{for} \quad \theta \geq \frac{c-s}{r-s} \\ \pi_{C1}(Q^*_{C1}) &= 0 \qquad \text{otherwise}\end{aligned} \tag{3}$$

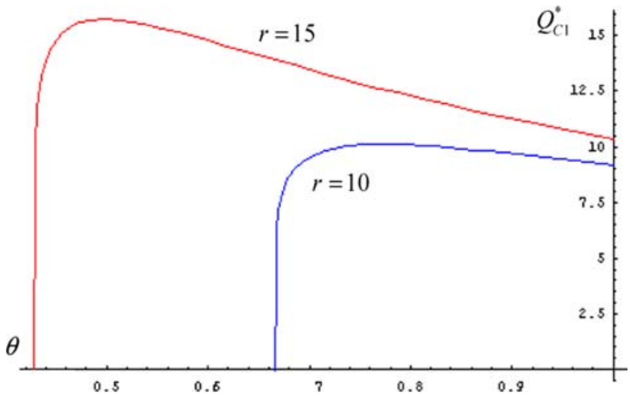

Fig. 1 Variation of Q^*_{C1} with θ for different values of r

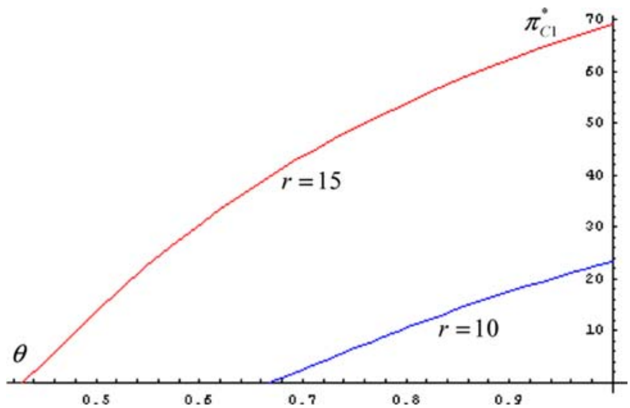

Fig. 2 Variation of π^*_{C1} with θ for different values of r

Further investigation leads to the following proposition:

Proposition 1 *For* $\theta \geq \dfrac{c-s}{r-s}$, *Model* 1 *is equivalent to a Newsvendor problem with a modified demand distribution having parameters* μ_{eq} *and* σ_{eq} *such that* $\mu_{eq} = \dfrac{\mu}{\theta}$, $\sigma_{eq} = \dfrac{\sigma}{\theta}$, *and equivalent modified unit costs* c_{eq}, s_{eq} *and* r_{eq} *such that* $c_{eq} = c$, $s_{eq} = s$ *and* $r_{eq} = r\theta + s(1-\theta)$.

The following figures, (Figs. 1 and 2) represent respectively the variation of Q^*_{C1} and π^*_{C1} with θ for different values of r for $c = 7$, $s = 1$, $\mu = 10$ and $\sigma = 2$:

Remark 1 Note that when θ decreases, the product availability decreases since the quantity that the customer has access to is θQ_{C1}. To remedy to the decrease of the product availability, the solution is to order more since the available to buy quantity θQ_{C1} is increasing in the ordering quantity. As a consequence Q_{C1} increases when θ decreases. But below a critical value of θ, ordering more to increase the product availability increases also the quantity which is not available to buy ($(1-\theta)Q_{C1}$) which will be discounted. So, below this critical value of θ, Q_{C1} decreases when θ decreases. For small values of θ $\left(\theta < \dfrac{c-s}{r-s}\right)$, the available to buy quantity (θQ_{C1}) is small. Even if a big quantity is ordered, the available to buy quantity remains small, so the trade-off between underage and overage penalties is established for $Q^*_{C1} = 0$. Concerning the expected profit, note that, as expected, it decreases when θ decreases as illustrated in Fig. 2.

3.2 Analysis of the decentralized uncoordinated scenario (DU1)

Under this scenario we assume that the manufacturer and the retailer are two independently owned and managed firms, where each party is trying to maximize his own profit. We analyze in this section the case where the two supply chain actors do not coordinate. We consider the wholesale contract: the manufacturer chooses the unit wholesale price w_{DU1} and after observing w_{DU1}, the retailer chooses the order quantity Q_{DU1}. Recall that both the manufacturer and the retailer can observe the error parameter θ and optimize their inventory systems by taking into account θ. The decision action of the manufacturer depends on the decision action of the retailer and vice versa. Game theory gives precious tools to determine these actions. In this paper, we are interested in a Stackelberg equilibrium where the manufacturer acts as a Stackelberg leader and offers a take-it or leave-it proposition to the retailer.

3.2.1 The retailer's problem

In Model 1, the retailer's profit function under a wholesale contract is similar to the profit function of the Centralized scenario of the same model (Model 1) with the exception that the retailer now pays a wholesale price w_{DU1} to the manufacturer whose unit production cost is still c. The expected profit for the retailer is also as follows:

$$\begin{aligned}\pi^R_{\text{DU1}}(Q_{\text{DU1}}, w_{\text{DU1}}) = (r - w_{\text{DU1}})\mu &\\ -(r - w_{\text{DU1}}) \int\limits_{x=\theta Q_{\text{DU1}}}^{+\infty} (x - \theta Q_{\text{DU1}}) f(x)\mathrm{d}x &\\ +(w_{\text{DU1}} - s) \int\limits_{x=0}^{Q_{\text{DU1}}} (\theta Q_{\text{DU1}} - x) f(x)\mathrm{d}x &\\ -(w_{\text{DU1}} - s) Q_{\text{DU1}}(1-\theta) &\end{aligned} \tag{4}$$

As shown in Sect. 3.1, for $\theta \geq \dfrac{w_{DU1} - s}{r - s}$, the optimal ordering quantity should verify:

$$F(\theta Q^*_{\text{DU1}}) = \frac{r\theta + (1-\theta)s - w_{\text{DU1}}}{(r-s)\theta} \tag{5}$$

For the case where $\theta \leq \dfrac{w_{\mathrm{DU1}} - s}{r - s}$, it is optimal for the retailer to not order because the trade-off between underage and overage penalties is established for an optimal ordering quantity equal to zero. In the rest of the paper, to be close to practical values of parameters, we assume that model parameters are such that an order is placed and only results pertaining to this situation will be developed. Estimates of practical values taken by θ can be found in empirical researches such as the investigation of Raman et al. (2001) that states that consumers of a leading retailer cannot find in average 16% of items in the stores because those items are misplaced. In our numerical examples we will assume that $0.8 \leq \theta \leq 1$.

3.2.2 *The manufacturer's problem*

The manufacturer has the wholesale price w_{DU1} as decision variable. He is able to anticipate the retailer's order for any wholesale price. As a consequence, the function $Q_{\mathrm{DU1}}(w_{\mathrm{DU1}})$ is deterministic for him. The manufacturer's problem then is to choose the wholesale price w_{DU1} that maximizes his expected profit $\pi^{M}_{\mathrm{DU1}}(w_{\mathrm{DU1}})$ which is given by:

$$\pi^{M}_{\mathrm{DU1}}(w_{\mathrm{DU1}}) = (w_{\mathrm{DU1}} - c) Q_{\mathrm{DU1}}(w_{\mathrm{DU1}}) \tag{6}$$

Theorem 2 *For Model* 1 *under an IGFR*[1] *demand distribution*

(a) *The optimum is reached for Q^*_{DU1}, such that*

$$1 - F(\theta Q^*_{\mathrm{DU1}}) - \theta Q^*_{\mathrm{DU1}} f(\theta Q^*_{\mathrm{DU1}}) = \frac{c - s}{r - s}\frac{1}{\theta}$$

(b) *The corresponding optimum wholesale price is*

$$w^*_{\mathrm{DU1}} = c + (r - s)\theta(\theta Q^*_{\mathrm{DU1}}) f(\theta Q^*_{\mathrm{DU1}})$$

(c) *The optimal expect profit of the manufacturer is*

$$\pi^{M*}_{\mathrm{DU1}} = (r - s)(\theta Q^*_{\mathrm{DU1}})^2 f(\theta Q^*_{\mathrm{DU1}})$$

(d) *The optimal expect profit of the retailer is*

$$\pi^{R*}_{\mathrm{DU1}} = (r - s) \int_{x=0}^{\theta Q^*_{\mathrm{DU1}}} x f(x) \mathrm{d}x$$

Proof cf Appendix 1 □

Theorem 2 enables us to identify some interesting properties:

[1] Increasing general failure rate.

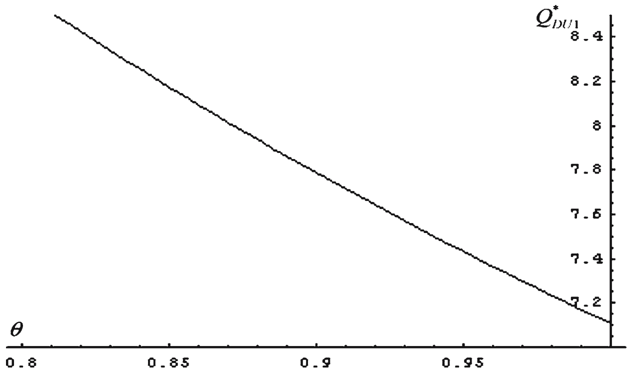

Fig. 3 Variation of Q^*_{DU1} with θ

Property 1 In Model 1 under a wholesale price contract:

(a) The manufacturer's optimal amount of product sold to the retailer Q^*_{DU1}
- increases as the retail price r and the salvage price s increase
- decreases as the unit production cost c increases

(b) The manufacturer's optimal wholesale price charged to the retailer w^*_{DU1}
- decreases as the retail price r and the salvage price s increase
- increases as the unit production cost c increases

For the case of a normally distributed demand, some interesting results concerning the variation of Q^*_{DU1}, w^*_{DU1} and π^{M*}_{DU1} with the error parameter θ can be deduced as proposed in the following Property:

Property 2 The impact of errors in the DU scenario of Model 1 is as follows:

(a) w^*_{DU1} decreases as θ decreases
(b) $\theta Q^*_{\mathrm{DU1}}$, decreases as θ decreases
(c) The manufacturer's expected optimal profit, π^{M*}_{DU1}, decreases as θ decreases
(d) The retailer's expected optimal profit, π^{R*}_{DU1}, decreases as θ decreases

Proof cf Appendix 2 □

Note that for reasonable values of model parameters, Q^*_{DU1}, increases as θ decreases. To get further insights, we consider an example where demand is normally distributed with parameters $\mu = 10$ and $\sigma = 2$, the unit production cost is $c = 7$, the unit selling price and the unit salvage price $r = 15$ and $s = 1$, respectively. Concerning the optimal ordering and wholesale price, figures below, represent respectively, the variation of Q^*_{DU1} and w^*_{DU1} with θ (Figs. 3, 4):

The following observations explain the variations of Q^*_{DU1} and w^*_{DU1} with θ:

- When $\theta = 1$, Q^*_{DU1} corresponds to the optimal ordering quantity of Model 0. As in the Centralized scenario, the optimal ordering quantity in Model 1 is more important than the one of Model 0 and increases as θ decreases (for reasonable values of model parameters). Such result is not surprising since the presence of errors decreases product's availability. Increasing the ordering quantity is the way to increase the available to buy quantity and to remedy to shelf unavailability.

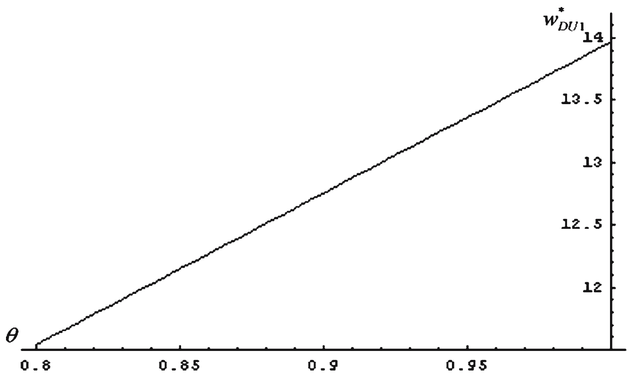

Fig. 4 Variation of w^*_{DU1} with θ

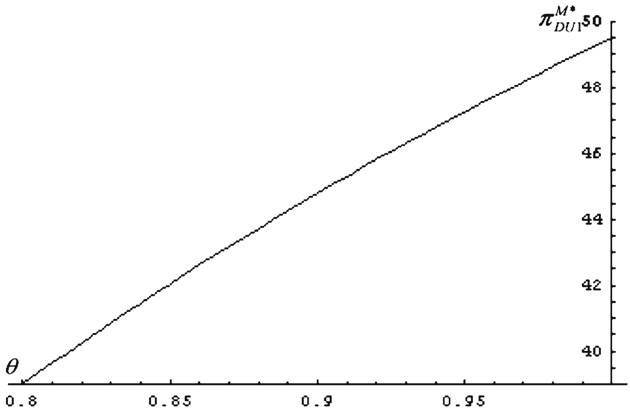

Fig. 5 Variation of π^{M*}_{DU1} with θ

- As a consequence of the last observation, the manufacturer's optimal wholesale price charged to the retailer in Model 1 is less important than the one in Model 0 and decreases as θ decreases.

Concerning the optimal expected profit achieved by each supply chain actor, Figs. 5 and 6 represent, respectively, the variation of π^{M*}_{DU1} and π^{R*}_{DU1} with θ:

As expected, the retailer suffers from the presence of errors in his store since his expected profit function decreases when θ decreases. As explained later, because of errors, the manufacturer's amount of product sold to the retailer increases. So, it is not unreasonable to expect that the inventory inaccuracy might have beneficial effects on the manufacturer expected profit. This is not true because the manufacturer should decrease the wholesale price charged to the retailer. As a consequence, as illustrated in Fig. 5, the manufacturer also suffers from inventory inaccuracy in the retailer's store.

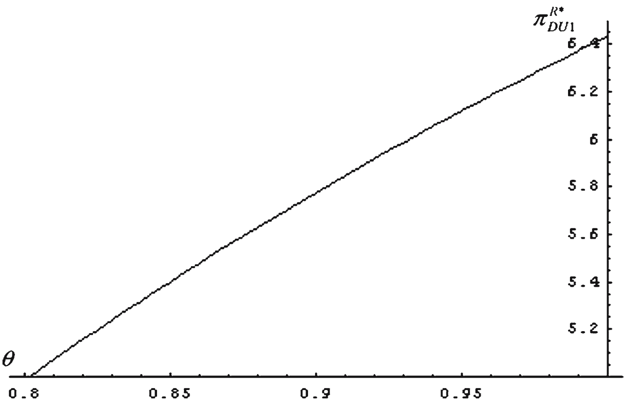

Fig. 6 Variation of π_{DU1}^{R*} with θ

3.2.3 Comparison between C1 and DU1

An important aspect to consider is the efficiency of the supply chain which measures how efficient the Decentralized Uncoordinated scenario performs in relation to the Centralized scenario. In the Decentralized Uncoordinated scenario, the outcomes are worse for all the parties involved (manufacturer, retailer, supply chain, and consumer) compared to the Centralized scenario, because in the Decentralized scenario both the retailer and the manufacturer independently try to maximize their own profits, i.e., they each try to get a margin. This effect is called "double marginalization" . The supply chain efficiency is defined as the ratio between the total supply chain profit in the Decentralized Uncoordinated scenario and the Centralized scenario profit.

For Model 1, the supply chain efficiency is given by:

$$\text{eff}_1 = \frac{\pi_{\text{DU1}}^{M*} + \pi_{\text{DU1}}^{R*}}{\pi_{C1}^{*}} \tag{7}$$

Figure 7 represents the variation of the supply chain efficiency with θ for $\mu = 10$, $\sigma = 2$, $c = 7$, $s = 1$ and $r = 15$. We note that for reasonable values of θ, this efficiency increases as θ decreases and this is somewhat surprising but can be explained as follows: as we have shown, Model 1 can be considered as an equivalent Newsvendor problem with modified demand distribution such that: $\mu_{\text{eq}} = \frac{\mu}{\theta}$, $\sigma_{\text{eq}} = \frac{\sigma}{\theta}$, and equivalent modified unit costs c_{eq}, s_{eq} and r_{eq} such that $c_{\text{eq}} = c$, $s_{\text{eq}} = s$ and $r_{\text{eq}} = r\theta + s(1-\theta)$. When θ decreases, both r_{eq} and w_{DU1}^{*} charged by the manufacturer to the retailer decrease. This induces a reduction of the double marginalization effect since values of r_{eq} and w_{DU1}^{*} are closer.

Remark 2 Based on Remark 1, analyzing the behaviour of the optimal ordering quantity with θ for decreasing values of θ permits us to deduce that there is a critical value of θ (the same critical value mentioned in Remark 1 which causes the change of slope in the variation of the ordering quantity) above which eff_1 decreases. For these values of θ, when θ takes decreasing values, the optimal ordering quantity decreases, w_{DU1}^{*}

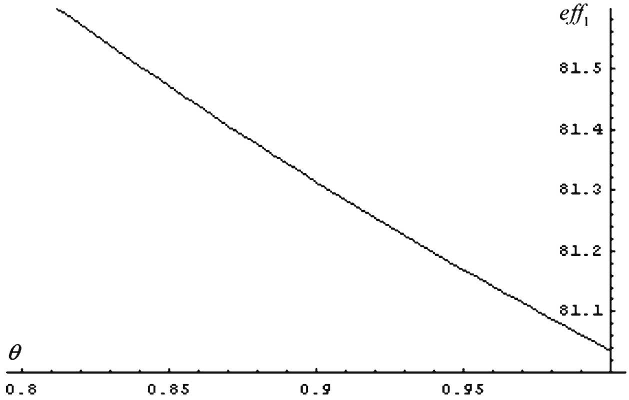

Fig. 7 Variation of eff_1 with θ

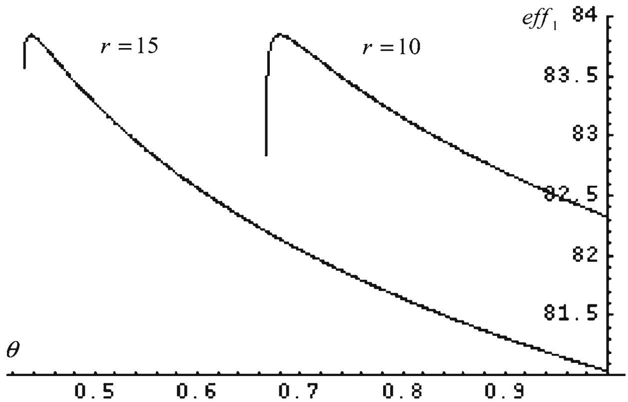

Fig. 8 Superior bound of eff_1

increases and as a consequence, the effect of the double marginalization increases, which in turn, makes the supply chain efficiency decrease with θ. This last behaviour enables also to show that the supply chain efficiency is bounded by a higher bound as it is show in Fig. 8.

3.3 Analysis of the decentralized coordinated scenario (DC1)

In this section, we analyze the Decentralized Coordinated scenario and how one can design contracts such that even though each supply chain actor acts out for self interest, the decentralized solution approaches the centralized optimal solution.

We consider a modified buy-back contract: as an incentive for the retailer to order more and move toward channel coordination, the manufacturer offers to buy back the unsold quantities of the available to buy quantity (the quantity which was on shelf and available to consumers during the selling period). We interpret the working of our modified buy-back contract such that the manufacturer pays $(b_{\mathrm{DC1}} - s)$ to each unsold unit of the available to buy quantity, and the retailer salvages the item for s. In other terms, at the end of the period, the manufacturer buys back units of products which were at their right place in the store. He also shares with the retailer the demand uncertainty risk but he stays indifferent with the inventory inaccuracy occurring in the retailer's store. With such a buy-back contract, the retailer is essentially getting a higher "salvage" value, b_{DC1}, for a fraction of the unsold goods, the other fraction (the non available to buy quantity) will continue to be discounted at the price s.

3.3.1 The retailer's problem

The retailer's expected profit function in Model 1 under our modified buy-back contract is given by:

$$\begin{aligned}\pi^R_{\mathrm{DC1}}(Q_{\mathrm{DC1}}) &= (r - w_{DC1})\mu - (r - w_{\mathrm{DC1}}) \int\limits_{x=\theta Q_{\mathrm{DC1}}}^{+\infty} (x - \theta Q_{DC1}) f(x)\mathrm{d}x \\ &\quad -(w_{\mathrm{DC1}} - b_{\mathrm{DC1}}) \int\limits_{x=-\infty}^{\theta Q_{\mathrm{DC1}}} (\theta Q_{\mathrm{DC1}} - x) f(x)\mathrm{d}x \\ &\quad -(w_{\mathrm{DC1}} - s) Q_{\mathrm{DC1}}(1-\theta) \end{aligned} \tag{8}$$

By using the same optimization method as in the Centralized scenario, we can show that the optimal ordering quantity Q^*_{DC1} and the optimal expected profit for the retailer, $\pi^R_{\mathrm{DC1}}(Q^*_{\mathrm{DC1}})$, should respectively satisfy:

$$F(\theta Q^*_{\mathrm{DC1}}) = \frac{(r-s)\theta - (w_{\mathrm{DC1}} - s)}{(r - b_{\mathrm{DC1}})\theta} = \frac{r_{\mathrm{eq}} - w_{\mathrm{DC1}}}{r_{\mathrm{eq}} - s_{\mathrm{eq}}}$$

$$where\ r_{\mathrm{eq}} = r\theta + (1-\theta)s \quad \text{and} \quad s_{\mathrm{eq}} = b\theta + (1-\theta)s \tag{9}$$

$$\pi^R_{\mathrm{DC1}}(Q^*_{\mathrm{DC1}}) = (r - b_{\mathrm{DC1}}) \int\limits_{x=0}^{\theta Q^*_{\mathrm{DC1}}} x f(x)\mathrm{d}x \tag{10}$$

It is straightforward to verify that the retailer's optimal ordering quantity and profit are increasing in b_{DC1} for a fixed wholesale price w_{DC1}.

3.3.2 The manufacturer's problem

With our modified buy-back contract, the expected profit of the manufacturer is given by:

$$\pi^M_{\mathrm{DC1}}(w_{\mathrm{DC1}}, b_{\mathrm{DC1}}) = (w_{\mathrm{DC1}} - c) Q_{\mathrm{DC1}} - (b_{\mathrm{DC1}} - s) \int\limits_{0}^{\theta Q_{\mathrm{DC1}}} F(x)\,\mathrm{d}x \tag{11}$$

The buy-back contract is completely determined by the 2-tuple $(w_{\mathrm{DC1}}, b_{\mathrm{DC1}})$, where w_{DC1} and b_{DC1} are the wholesale price and the buy-back price, respectively. The following proposition states condition on model parameters under which channel coordination is realized:

Theorem 3 *There is a 2-tuple $(w_{\mathrm{DC1}}(\varepsilon), b_{\mathrm{DC1}}(\varepsilon))$ that is able to coordinate the decentralized scenario $w_{\mathrm{DC1}}(\varepsilon) = r_{\mathrm{eq}} - \varepsilon$ and $b_{\mathrm{DC1}}(\varepsilon) = r - \varepsilon\frac{r-s}{r_{\mathrm{eq}}-c}$ where $r_{\mathrm{eq}} = r\theta + s(1-\theta)$ and $\varepsilon \in (0, r_{\mathrm{eq}} - c)$*

(a) *The retailer orders the optimal solution of the Centralized scenario and system profit is also equal to the Centralized scenario profit.*
(b) *Retailer profit is increasing in ε. Specially $\pi^{R*}_{\mathrm{DC1}}(w_{\mathrm{DC1}}(\varepsilon), b_{\mathrm{DC1}}(\varepsilon)) = \frac{\varepsilon}{r_{\mathrm{eq}}-c}\pi^*_{C1}$.*
(c) *Manufacturer profit is decreasing in ε. Specially $\pi^{M*}_{\mathrm{DC1}}(w_{\mathrm{DC1}}(\varepsilon), b_{\mathrm{DC1}}(\varepsilon)) = (1 - \frac{\varepsilon}{r_{\mathrm{eq}}-c})\pi^*_{\mathrm{C1}}$.*

Proof cf Appendix 3 □

The parameter ε governs the distribution of market power and determines how the benefit achieved by coordination is shared between supply chain actors. We notice that when θ decreases, the retailer gets a higher sharing of the total supply chain profit. The following property states the variation of the parameters of the buy back contract with the error parameter θ:

Property 3 w_{DC1} and b_{DC1} charged to the retailer decrease when the error parameter θ decreases.

Proof The proof follows directly by observing that $\frac{\partial w_{\mathrm{DC1}}(\varepsilon)}{\partial\theta} \geq 0$ and $\frac{\partial b_{\mathrm{DC1}}(\varepsilon)}{\partial\theta} \geq 0$. □

4 Analysis of Model 2

In Model 2, the RFID technology is deployed in order to eliminate misplacement errors in the store. We will assume that the cost associated with the implementation of this technology consists in RFID tags embedded to each unit of product individually, at a certain tag cost t. For simplification, the fixed costs of investments necessary to implement the technology (such as reader systems cost, infrastructure costs, basic application integration costs, maintenance and support costs and overhead costs) are deliberately not part of our model.

In the Centralized scenario the notion of sharing the cost t does not make sense. In the Decentralized Uncoordinated scenario, under a wholesale contract we will assume that the manufacturer will pay the whole tag cost t. We will show that the notion of sharing the tag cost will not influence the optimal solution. Indeed, the manufacturer will simply adjust the wholesale price in order to include to it his part pertaining to the RFID tag cost. The same remark holds also under the Decentralized Coordinated scenario.

When the RFID technology is deployed, the unit production cost is no longer c but $c + t$: the optimization of Model 2 under each scenario is therefore a modified

Table 2 Results pertaining to each scenario under Model 2

Scenario	Result
C2	$$F(Q^*_{C2}) = \frac{r-(c+t)}{r-s}$$ $$\pi_2(Q^*_{C2}) = (r-s)\int_{x=0}^{Q^*_{C2}} xf(x)dx$$
DU2	**Theorem 4** *For Model 2 under an IGFR demand distribution* (a) *The optimum is reached for Q^*_{DU2}, such that*: $$1-F(Q^*_{\mathrm{DU2}}) - Q^*_{\mathrm{DU2}} f(Q^*_{\mathrm{DU2}}) = \frac{c-s+t}{r-s}$$ (b) *The corresponding optimum wholesale price is*: $$w^*_{\mathrm{DU2}} = c+t+(r-s)Q^*_{\mathrm{DU2}} f(Q^*_{\mathrm{DU2}})$$ (c) *The optimum expect profit of the manufacturer is*: $$\pi^{M*}_{\mathrm{DU2}} = (r-s)(Q^*_{\mathrm{DU2}})^2 f(Q^*_{\mathrm{DU2}})$$ (d) *The optimum expect profit of the retailer is*: $$\pi^{R*}_{\mathrm{DU2}} = (r-s)\int_{x=0}^{Q^*_{\mathrm{DU2}}} xf(x)\,\mathrm{d}x$$
DU2	**Theorem 5** *Suppose that the manufacturer offers a contract $(w_{\mathrm{DC2}}(\varepsilon), b_{\mathrm{DC2}}(\varepsilon))$ for $\varepsilon \in (0, r-c-t)$ where $w_{DC2}(\varepsilon) = r-\varepsilon$ and $b_{DC2}(\varepsilon) = r - \varepsilon\frac{r-s}{r-(c+t)}$* : (a) *The retailer order the optimal solution of the Centralized Scenario and system profit is also equal to the Centralized Scenario profits* (b) *Retailer profit is increasing in ε. Specially $\pi^{R*}_{\mathrm{DC2}}(w_{\mathrm{DC2}}(\varepsilon), b_{\mathrm{DC2}}(\varepsilon)) = \frac{\varepsilon}{r-(c+t)}\pi^*_{C2}$* (c) *Manufacturer profit is decreasing in ε. Specially $\pi^{M*}_{\mathrm{DC2}}(w_{\mathrm{DC2}}(\varepsilon), b_{\mathrm{DC2}}(\varepsilon)) = (1-\frac{\varepsilon}{r-(c+t)})\pi^*_{C2}$*

Newsvendor problem with a production cost equal to $c+t$. The following table summarizes the main results pertaining to each scenario. These results are already available in the literature: Khouja (1999) for the Centralized scenario, Larivière and Porteus (2001) for the Decentralized Uncoordinated scenario and Pasternack (1985) for the Decentralized Coordinated scenario (Table 2).

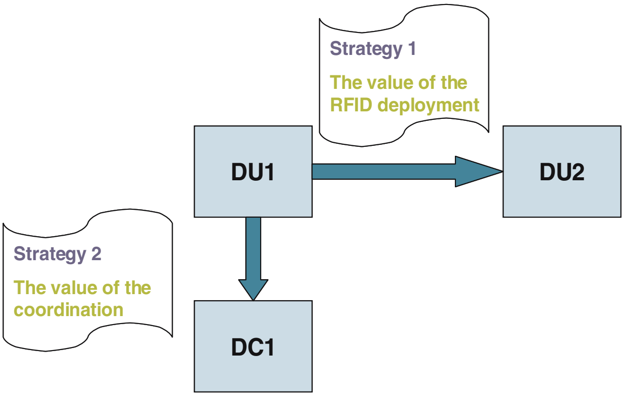

Fig. 9 Strategy 1 versus Strategy 2

As expected, even if we assumed that the manufacturer pays the tag cost, he will adjust the wholesale price in order to include this additional cost. This is why the notion of sharing the tag price is not relevant. To focus on this result, let consider two settings where the manufacturer pays a fraction $\alpha_1 t$ ($\alpha_2 t$, respectively) and the retailer pays the rest $(1-\alpha_1)t$ ($(1-\alpha_2)t$, respectively) in the first (second, respectively) setting. We can easily show that $[w^*_{\mathrm{DU2}}]_{\alpha_2} - [w^*_{\mathrm{DU2}}]_{\alpha_1} = (\alpha_2 - \alpha_1)t$. As a consequence $[Q^*_{\mathrm{DU2}}]_{\alpha_2} = [Q^*_{\mathrm{DU2}}]_{\alpha_1}$ which ensures that $[\pi^{M*}_{\mathrm{DU2}}]_{\alpha_2} = [\pi^{M*}_{\mathrm{DU2}}]_{\alpha_1}$ and $[\pi^{R*}_{\mathrm{DU2}}]_{\alpha_2} = [\pi^{R*}_{\mathrm{DU2}}]_{\alpha_1}$. The same reasoning holds for the case of the buyback contract.

5 Strategies reducing the impact of errors

The case of a centralized supply chain: In an earlier paper (Rekik et al. 2006), we have shown that in a centralized supply chain it exists a critical unit tag price t_{cr} such that i) for $t \geq t_{\mathrm{cr}}$ the implementation of the RFID technology is not beneficial ii) for $t \leq t_{\mathrm{cr}}$ the implementation of the RFID technology yields a positive benefit. This critical value of t is the solution of the equation $\pi_{C2}(Q^*_{C2}) = \pi_{C1}(Q^*_{C1})$ and is given by $t_{\mathrm{cr}} = \dfrac{1-\theta}{\theta}(c-s)$.

The case of a decentralized supply chain: In this section, we focus on the case of a decentralized supply chain and present two strategies that may enable to both actors to reduce the impact of misplacement errors. We assume that they initially manage their inventory under the Decentralized Uncoordinated scenario. Our aim is to compare the performance of the following two strategies (Fig. 9):

– Strategy 1: introducing the RFID technology while being initially in a Decentralized Uncoordinated supply chain setting (i.e. the transition from DU1 to DU2)
– Strategy 2: coordinating the supply chain in presence of errors (i.e. the transition from DU1 to DC1)

Throughout this section, we consider a normally distributed demand. Note that our analysis can be extended to deal with other distributions.

5.1 Strategy 1: introduction of the RFID technology

This section focuses on the comparison between Model 1 and Model 2 under a wholesale contract, our aim being to answer the question "Under which circumstances both the retailer and the manufacturer will be motivated to deploy the RFID technology?"

Concerning the comparison between Q^*_{DU1} and Q^*_{DU2}, the following proposition should be made:

Proposition 2 *Under a wholesale price contract, if* $t \le (c-s)\frac{1-\theta}{\theta}$ *we have* $\theta Q^*_{\mathrm{DU1}} \le Q^*_{\mathrm{DU2}}$.

Proof The proof follows by observing that $\frac{c-s}{r-s}\frac{1}{\theta} \le \frac{c-s+t}{r-s}$ if $t \le (c-s)\frac{1-\theta}{\theta}$. Using the fact that both θQ^*_{DU1} and Q^*_{DU2} are smaller than μ and the fact that $H(y) = 1 - F(y) - yf(y)$ is decreasing in y for $y \le \mu$ (cf Appendix 2), the result can be deduced directly. □

As a consequence of the last result we can show that:

Proposition 3 *Under a wholesale price contract, if* $t \le (c-s)\frac{1-\theta}{\theta}$ *we have* $\pi^{M*}_{\mathrm{DU1}} \le \pi^{M*}_{\mathrm{DU2}}$.

Proof the proof is obtained by using the result of the previous proposition and by observing that $\theta Q^*_{\mathrm{DU1}} \le Q^*_{\mathrm{DU2}} \le \mu$ *and the fact that* $f(x)$*is increasing in* x *for* $x \le \mu$. □

As a consequence we deduce that $t \le (c-s)\frac{1-\theta}{\theta}$ is a sufficient condition to make the manufacturer interested in deploying the RFID technology in order to remedy to the inventory inaccuracy in the retailer's store. The following proposition answers the question "Is this condition makes the RFID technology interesting for the retailer also?"

Proposition 4 *Under a wholesale price contract, if* $t \le (c-s)\frac{1-\theta}{\theta}$ *we have* $\pi^{R*}_{\mathrm{DU1}} \le \pi^{R*}_{\mathrm{DU2}}$.

Proof The proof follows by using the fact that when $t = (c-s)\frac{1-\theta}{\theta}$ we have $\pi^{R*}_{\mathrm{DU1}} = \pi^{R*}_{\mathrm{DU2}}$ and the fact that the optimal expected profit of the retailer in Model 2 is decreasing in the tag price t. □

The following proposition summarizes the condition under which both the retailer and the manufacturer are interested in deploying the RFID technology:

Theorem 6 *Under a wholesale price contract,* $t \le t_{\mathrm{cr}} = \frac{1-\theta}{\theta}(c-s)$ *is a sufficient condition to make the retailer and the manufacturer choose the deployment of RFID technology.*

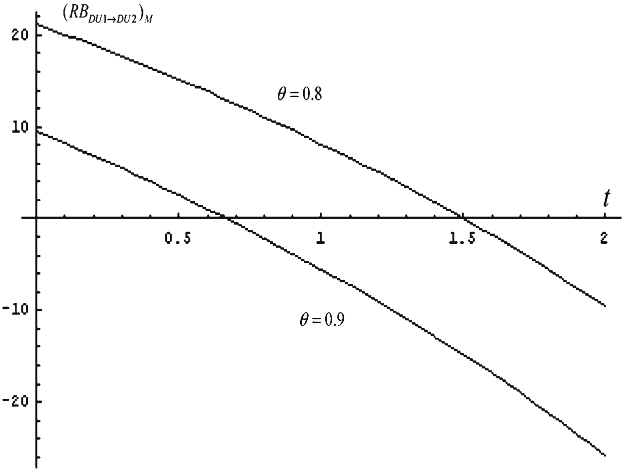

Fig. 10 Variation of $(RB_{\mathrm{DU1-DU2}})_M$ with t

Proof The proof follows by using results of Propositions 3 and 4. □

It is important to notice here that the critical unit tag cost provided in the last theorem is the same as the one presented in the Centralized scenario. This confirms our finding that the notion of sharing the tag cost between the supply chain actors under the DU scenario does not affect optimal solutions.

To quantify the relative benefits achieved by the retailer and the manufacturer stemming from the deployment of the RFID technology, we introduce the following two ratios:

$$(RB_{\mathrm{DU1}\to\mathrm{DU2}})_M = \frac{\pi^{M*}_{\mathrm{DU2}} - \pi^{M*}_{\mathrm{DU1}}}{\pi^{M*}_{\mathrm{DU1}}} \times 100 \tag{12}$$

$$(RB_{\mathrm{DU1}\to\mathrm{DU2}})_R = \frac{\pi^{R*}_{DU2} - \pi^{R*}_{\mathrm{DU1}}}{\pi^{R*}_{\mathrm{DU1}}} \times 100 \tag{13}$$

which measure the relative benefit achieved, respectively, by the manufacturer and the retailer from applying Strategy 1.

For our numerical example ($\mu = 10$, $\sigma = 2$, $c = 7$, $r = 15$ and $s = 1$) and for two values of θ such that $\theta = 0.8$ and $\theta = 0.9$, the following figures represent, respectively, the benefit that the manufacturer and the retailer achieve by applying Strategy 1 as a function of the tag cost:

As can be observed in figures above (Figs. 10, 11): for an error parameter such that $\theta = 0.8$, it is beneficial for the two supply chain actors to deploy the RFID technology if the tag price is under the critical value $t_{cr} = 1.5$. It is also important to note that the critical tag price decreases when the error parameter increases. For high values of θ, the tag price should be small in order to be adopted by the supply chain actors. We notice also that the critical tag price depends on the value of the product which it will

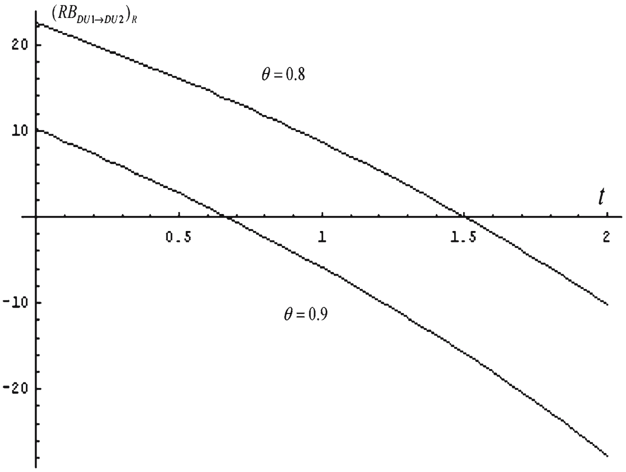

Fig. 11 Variation of $(RB_{\mathrm{DU1-DU2}})_R$ with t

be affixed to. For a small value of the production cost of the product, the tag price should be very small to be adopted.

5.2 Strategy 2: coordination of the supply chain

In order to quantify the benefit achieved by the retailer and the manufacturer by coordinating the channel, we introduce the following two ratios:

$$(RB_{\mathrm{DU1}\to\mathrm{DC1}})_M = \frac{\pi_{\mathrm{DC1}}^{M*} - \pi_{\mathrm{DU1}}^{M*}}{\pi_{\mathrm{DU1}}^{M*}} * 100 \tag{14}$$

$$(RB_{\mathrm{DU1}\to\mathrm{DC1}})_R = \frac{\pi_{\mathrm{DC1}}^{R*} - \pi_{\mathrm{DU1}}^{R*}}{\pi_{\mathrm{DU1}}^{R*}} * 100 \tag{15}$$

An important issue to be considered in designing our modified buy back contract concerns the flexibility of the contract i.e., both manufacturer and retailer should obtain a profit higher than the one they will obtain without using this contract. Otherwise, the supply chain actors would not be prompted to adopt the contract. In order to ensure that both supply chain actors coordinate, the following proposition states a condition on ε which enables a positive benefit for the two supply chain actors:

Proposition 5

(a) *For a given θ, both the manufacturer and the retailer achieve a positive benefit by applying Strategy* 2 *for ε such that $\varepsilon_{\min} \le \varepsilon \le \varepsilon_{\max}$ where $\varepsilon_{\min} = \frac{\pi_{DU1}^{R*}}{\pi_{C1}^{*}}(r_{\mathrm{eq}} - c)$ and $\varepsilon_{\max} = \frac{\pi_{C1}^{*} - \pi_{\mathrm{DU1}}^{M*}}{\pi_{C1}^{*}}(r_{\mathrm{eq}} - c)$*

(b) *The lenght of interval in which all supply chain actors are interested in applying Strategy* 2, *$\varepsilon_{\max} - \varepsilon_{\min} = (1 - \mathrm{eff}_1)(r_{\mathrm{eq}} - c)$, decreases when θ decreases*

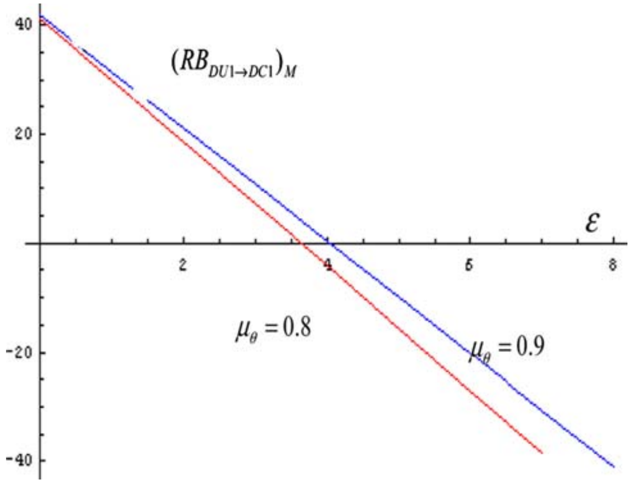

Fig. 12 Variation of $(RB_{DU1-DC1})_M$ with ε

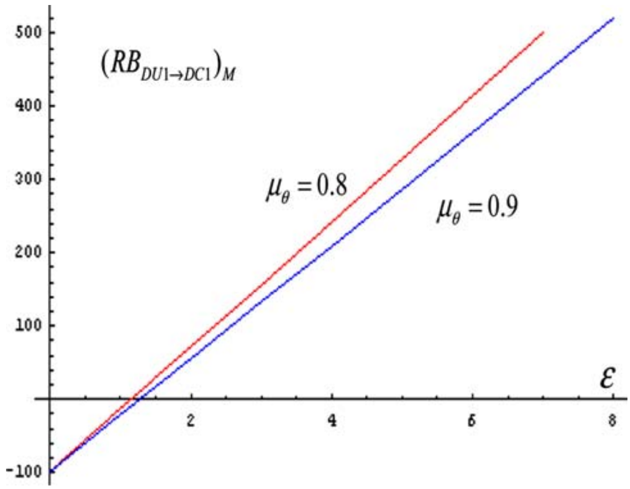

Fig. 13 Variation of $(RB_{DU1-DC1})_R$ with ε

Proof (a) $\varepsilon_{\min}$ and $\varepsilon_{\max}$ are derived by solving $(RB_{\text{DU1}\to\text{DC1}})_R = 0$ and $(RB_{\text{DU1}\to\text{DC1}})_M = 0$ respectively. (b) follows by observing that when θ decreases, eff_1 increases and r_{eq} decreases. □

For our numerical example ($\mu = 10, \sigma = 2, c = 7, r = 15$ and $s = 1$) and for two values of the error parameter such that $\theta = 0.8$ and $\theta = 0.9$, the following figures represent respectively the benefit that the manufacturer and the retailer achieve by applying Strategy 2 as a function of the market power ε (Figs. 12, 13):

Table 3 The benefits in each strategy

	Strategy 1	Strategy 2
Manufacturer	$B_1^M = \pi_{\mathrm{DU2}}^{M*} - \pi_{\mathrm{DU1}}^{M*}$	$B_2^M = \pi_{\mathrm{DC1}}^{M*} - \pi_{\mathrm{DU1}}^{M*}$
Retailer	$B_1^R = \pi_{\mathrm{DU2}}^{R*} - \pi_{\mathrm{DU1}}^{R*}$	$B_2^R = \pi_{\mathrm{DC1}}^{R*} - \pi_{\mathrm{DU1}}^{R*}$
Supply chain	$B_1^{\mathrm{SC}} = B_1^M + B_1^R$	$B_2^{\mathrm{SC}} = B_2^M + B_2^R$

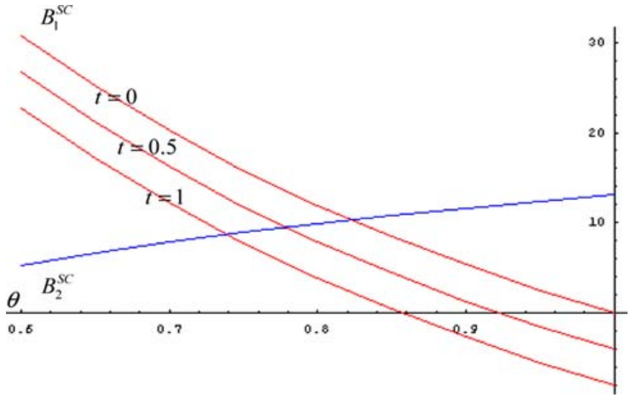

Fig. 14 Variation of B_1^{SC} and B_2^{SC} with θ for different values of t

5.3 Comparison between strategies

5.3.1 Evaluation of the best strategy for the entire supply chain

We consider a numerical example where $\mu = 10$, $\sigma = 2$, $c = 7$, $r = 15$ and $s = 1$ with the additional hypothesis that t may take three possible values (0, 0.5, 1) which represents respectively (0, 7, 14) % of the unit cost of production of the product. The question we consider now is "*What is the best strategy that will be followed by a decentralized supply chain where the store is subject to misplacement errors?*"

In order to answer this question, we proceed in two steps. We first analyze the best strategy for the entire supply chain. Then, we consider the manufacturer and the retailer as individual actors. The following table represents benefits that would be achieved by supply chain actors in each strategy (Table 3):

Figure 14 represents the variation of B_1^{SC} and B_2^{SC} with θ for different value of t.

The following observations can be made:

- Variation of B_1^{SC} with θ: when $\theta = 1$, deploying the RFID technology is not necessary and may lead to a negative gain for both supply chain actors because of the additional tag cost. When θ decreases (i.e. more errors in the system) the benefit achieved by the RFID technology is more important for both the manufacturer and the retailer and as a consequence for the entire supply chain. If shelf availability is poor, the RFID technology is justified economically (Fig. 14).

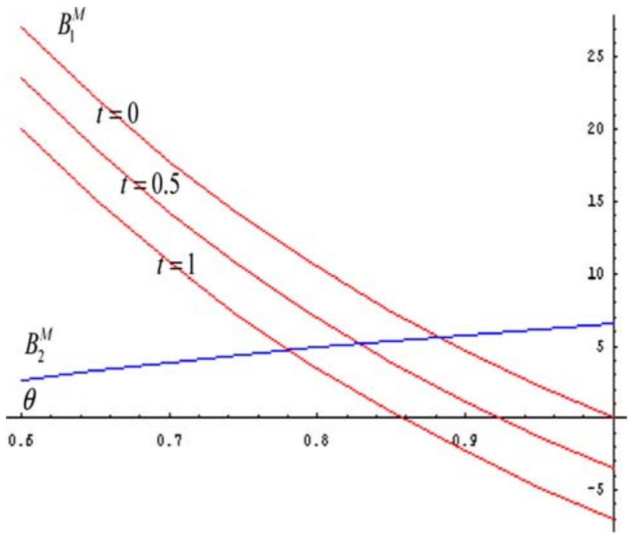

Fig. 15 Variation of B_1^M and B_2^M with θ

- Variation of B_2^{SC} with θ: as explained previously, errors affect the efficiency of the Decentralized Uncoordinated scenario. When θ decreases, eff_1 increases and as a consequence, the total gain, B_2^{SC}, achieved by coordinating the channel decreases.
- For a given tag price t, there exists a critical value of θ, $\theta_{\text{cr}}^{\text{SC}}$, which solves $B_1^{\text{SC}} = B_2^{\text{SC}}$, such that if $\theta \geq \theta_{\text{cr}}^{\text{SC}}$ deploying Strategy 2 is better than Strategy 1 for the entire supply chain, otherwise Strategy 1 is better. As intuitively expected, this critical value of θ decreases when t increases.

5.3.2 Evaluation of the best strategy for individual supply chain actors

By considering the retailer and the manufacturer as individual actors, we focus on the benefit that each supply chain actor achieves in the two strategies. In order to simplify the analysis, we assume that when the supply chain is coordinated (i.e. Strategy 2 is applied), the corresponding power market ε is chosen such that the total benefit achieved by channel coordination is equitably shared between the two supply chain actors. In other words, ε is chosen such that $\varepsilon = \dfrac{\varepsilon_{\max} + \varepsilon_{\min}}{2}$ for Strategy 2. For the same values of system parameters, applying either Strategy 1 or Strategy 2 will lead to the following figures (Figs. 15, 16):

The following observations can be made in order to answer the main research question of this section:

- There exists a critical value of θ, θ_{cr}^M (resp. θ_{cr}^R) which solves $B_1^M = B_2^M$ (resp. $B_1^R = B_2^R$), enabling to the manufacturer (retailer) to choose the best strategy between Strategy 1 and Strategy 2. If $\theta \geq \theta_{\text{cr}}^M$ (resp. $\theta \geq \theta_{\text{cr}}^R$) Strategy 2 is better than Strategy 1 for the manufacturer (resp. the retailer)

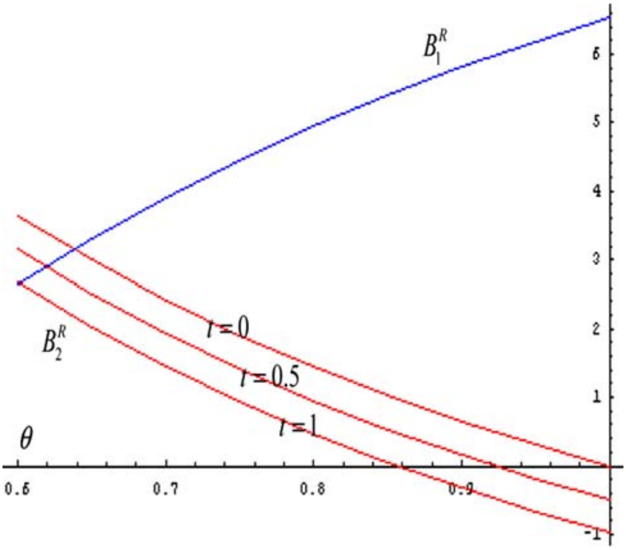

Fig. 16 Variation of B_1^R and B_2^R with θ

- It is important to notice that the critical value $\theta_{\text{cr}}^{\text{SC}}$ derived above does not enable to both supply chain actors to choose one of the strategies presented before. Even if the gain achieved by channel coordination (Strategy 2) is supposed to be equitably shared between them, the benefit achieved by the deployment of the RFID technology (Strategy 1) is not the same for the two supply chain actors. As illustrated on Fig. 15, the manufacturer profits more from the RFID technology than the retailer
- In this case, the manufacturer may have some difficulty in getting the retailer to implement RFID. One way to overcome this issue that can be observed in practice is to identify an appropriate value of ε when designing the buy-back contract so that both the manufacturer and the retailer will be interested in deploying the RFID technology at the same time. As illustrated on Figs. 17 and 18, for a given tag cost, there exists a critical value of ε which makes $\theta_{\text{cr}}^{R} = \theta_{cr}^{\text{M}}$. As a consequence, readjusting the parameter ε is a lever to bring the critical θ for the retailer closer up to the critical one for the manufacturer.

6 Conclusion

In this paper, we have presented an analytical model of a single-period store inventory subject to misplacement errors. We have compared different models for different scenarios, our objective being to compare the performance of actions enabling to reduce inventory inaccuracies stemming from misplacement type errors. The deployment of the RFID technology and the coordination of the supply chain are the specific actions we focused on. We have identified situations for which each strategy is the best to choose depending on the structure of the supply chain (centralized or decentralized) and values that system parameters take (error rate, RFID tag cost, . . .).

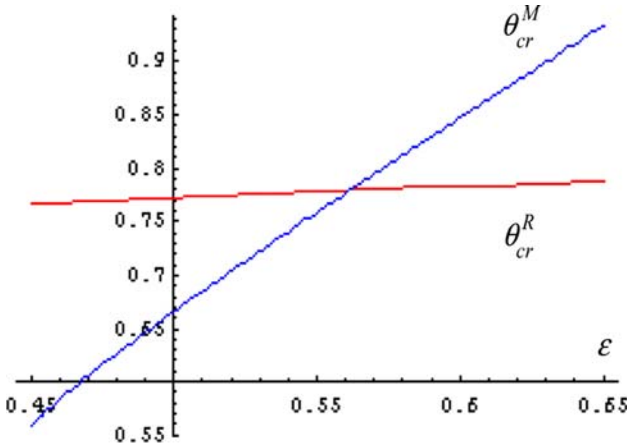

Fig. 17 Variations of $\theta_{\rm cr}^{R}$ and $\theta_{\rm cr}^{M}$ with ε, t = 0.5

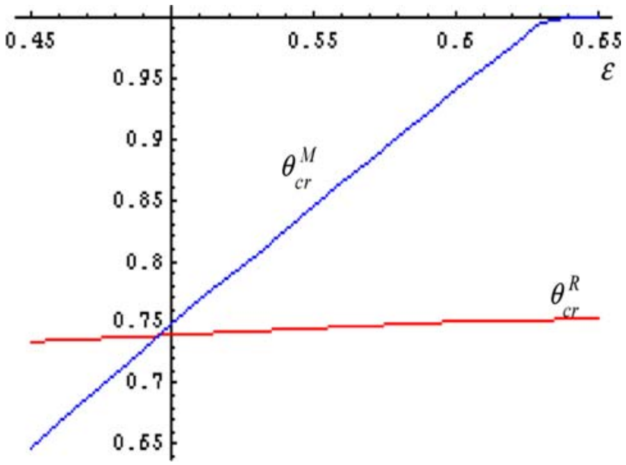

Fig. 18 Variations of $\theta_{\rm cr}^{R}$ and $\theta_{\rm cr}^{M}$ with ε, t = 1

Concerning the modeling framework, we have assumed that all products which are in the store (on shelves or misplaced) are sold at a salvage price s at the end of the period. Our model can be extended to include other types of errors leading to inaccuracies. In order to do this, we can introduce the unit price s_c ($s_c \le s$) that represents the additional cost that the retailer will inincuro find a misplaced product at the end of the period. In other terms, when a misplaced product is found at the end of the period, his unit salvage price is no more equal to s but to $(s - s_c)$. The introduction of the cost parameter s_c enables to consider all types of errors. Indeed, the case of errors such as theft or perishment where the quantity which is not available to buy is lost at the end of the period corresponds to a particular case where $s_c = s$.

Another important issue we are working on is to consider other alternatives to RFID enabling to reduce the inaccuracy problem. One could envisage a periodic inspection of the inventory (for example some products are counted one day, other products another day and so on). Such an alternative can be interesting in a multi period framework where the frequency of inspection and the order policy can be optimized.

Appendix 1

Using the fact that Q_{DU1} is such that:

$$F(\theta Q_{\text{DU1}}) = 1 - \frac{w_{\text{DU1}} - s}{r - s}\frac{1}{\theta}$$

By using the inverse of $Q_{\text{DU1}}(w_{\text{DU1}})$ which is given by:

$$w_{\text{DU1}} = (r_{\text{eq}} - s)(1 - F(\theta Q_{\text{DU1}})) + s$$

where $r_{\text{eq}} = r\theta + (1-\theta)s$. The expected profit of the manufacturer is also given by:

$$\begin{aligned}\pi^M_{\text{DU1}}(w_{\text{DU1}}) &= (w_{\text{DU1}} - c)Q_{\text{DU1}}(w_{DU1}) \\ &= [((r_{\text{eq}} - s)(1 - F(\theta Q_{\text{DU1}})) - (c - s))(\theta Q_{\text{DU1}})]\frac{1}{\theta}\end{aligned}$$

If we consider an IGFR distribution of the demand and the change of variable $Q'_{\text{DU1}} = \theta Q_{\text{DU1}}$, the rest of the proof follows directly by using Theorem 1 in Larivière and Porteus (2001).

Appendix 2

For the case of a normally distributed demand (which is IGFR) with parameter μ and σ, we fully describe in this appendix the function $H(y) = 1 - F(y) - yf(y)$ used throughout this paper in order to derive the optimal ordering quantity under the Decentralized Uncoordinated Scenario.

First it is important to show that $H(\mu) \leq 0$. In fact $H(\mu) = \frac{1}{2}\left(1 - \frac{\mu}{\sigma}\sqrt{\frac{2}{\pi}}\right)$ is negative since demand parameters are such that the coefficient of variation $cv = \frac{\sigma}{\mu} \leq \sqrt{\frac{2}{\pi}} \approx 0.8$.

Let now analyze the sens of variation of $H(y)$. The first derivative of $H(y)$ for a normally distributed demand is given by:

$$H'(y) = \frac{f(y)}{\sigma^2}(y^2 - \mu y - 2\sigma^2)$$

The first derivative is equal to zero for $y_1 = \frac{\mu - \sqrt{\mu^2 + 8\sigma^2}}{2} \leq 0$ and $y_2 = \frac{\mu + \sqrt{\mu^2 + 8\sigma^2}}{2} \geq \mu$. As a consequence we can conclude that $H(y)$ is decreasing in y for $y \in [0, y_2]$ where $y_2 \geq \mu$. In the other hand we can easily verify that $\lim_{y\to+\infty} H(y) = 0^-$.

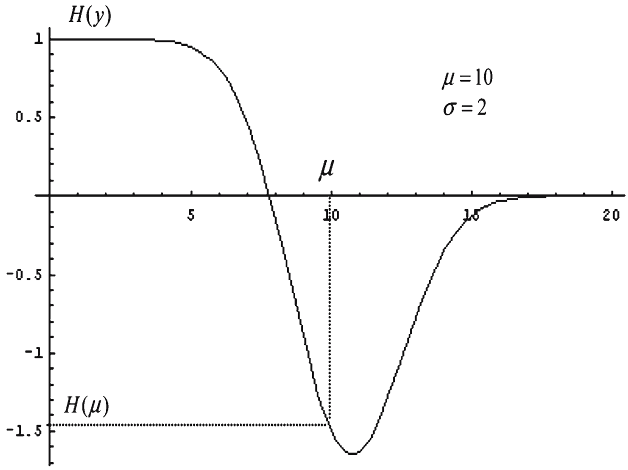

Fig. 19 Variation of $H(y)$ with y

The following figure represents the variation of $H(y)$ with y for $\mu = 10$ and $\sigma = 2$ (Fig. 19).

Combining the fact that $H(y)$ is decreasing in y for $y \in [0, y_2]$ where $y_2 \geq \mu$ and the fact that $H(\mu) \leq 0$ enable us to deduce some important results used throughout the paper. Specially the fact that Q^*_{DU0} which solves $H(Q^*_{\mathrm{DU0}}) = \frac{c-s}{r-s}$ (with $\frac{c-s}{r-s} \geq 0$) is such that:

Result 1 $Q^*_{\mathrm{DU0}} \leq \mu$

Two other important results concerning Q^*_{DU1}, which solves $H(\theta Q^*_{\mathrm{DU1}}) = \frac{1}{\theta}\frac{c-s}{r-s}$ are also deduced:

Result 2 $\theta Q^*_{\mathrm{DU1}} \leq Q^*_{\mathrm{DU0}} \leq \mu.$

Result 3 $\theta Q^*_{\mathrm{DU1}}$ decreases as θ decreases

By using the two last results and the fact the $f(x)$ is is increasing in x for $x \leq \mu$, the following results are deduced:

Result 4 $w^*_{\mathrm{DU1}} = c + (r-s)\theta(\theta Q^*_{\mathrm{DU1}}) f(\theta Q^*_{\mathrm{DU1}})$ decreases as θ decreases

Result 5 $\pi^{M*}_{\mathrm{DU1}} = (r-s)(\theta Q^*_{\mathrm{DU1}})^2 f(\theta Q^*_{\mathrm{DU1}})$ decreases as θ decreases

Result 6 $\pi^{R*}_{\mathrm{DU1}} = (r-s)\displaystyle\int_{x=0}^{\theta Q^*_{\mathrm{DU1}}} x f(x) dx$ decreases as θ decreases

Appendix 3

Observe that for all ε: $\dfrac{(r-s)\theta - (w_{\mathrm{DC1}} - s)}{(r - b_{\mathrm{DC1}})\theta} = 1 - \dfrac{c-s}{r-s}\dfrac{1}{\theta}$. The retailer faces the same critical fractile as the Centralized scenario and thus orders the same amount.

To determine retailer expected profit we have:

$$\pi^{R*}_{DC1}(w_{DC1}(\varepsilon), b_{DC1}(\varepsilon)) = (r - b_{DC1}) \int\limits_{x=0}^{\theta Q^*_{DC1}} xf(x)$$

$$= \varepsilon \frac{r - s}{r_{eq} - c} \int\limits_{x=0}^{\theta Q^*_{C1}} xf(x)$$

$$= \frac{\varepsilon}{r_{eq} - c} \pi^*_{C1}$$

Total system profit is equal to π^*_{C1} so, $\pi^{M*}_{\mathrm{DC1}}(w_{\mathrm{DC1}}(\varepsilon), b_{\mathrm{DC1}}(\varepsilon)) = \left(1 - \frac{\varepsilon}{r_{\mathrm{eq}} - c}\right) \pi^*_{C1}$

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